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COMPARISONS OF A STANDARD GALAXY MODEL WITH STELLAR OBSERVATIONS IN FIVE FIELDS

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ABSTRACT

Modern data on the distribution of stellar colors and on the number of stars as a function of apparent magnitude in five directions in the Galaxy are analyzed. Observational studies used are those of King; Koo and Kron; Kron, McLaughlin; Ratnatunga; Reid and Gilmore; Tritton and Morton; Tyson and Jarvis; and Weistrop. All of the data are described well by a two-component model with an exponential disk and a de Vaucouleurs spheroid. The observations are used to determine galactic parameters. For example, the scale height of disk dwarfs is found to be 325 ± 50 pc for $5 \leq M_V \leq 13.5$. Fluctuations in the average volume density of disk dwarfs are $\lesssim 15\%$. The dip in the luminosity function of nearby stars at $M_V = 7$ that was described by Wielen exists outside the solar vicinity. The spheroid axis ratio ($b/a = 0.80^{+0.20}_{-0.05}$), the normalization at the solar position (spheroid/disk ≈ 0.002), and the fraction of stars that are giants and subgiants are all inferred from the data with the aid of the standard galaxy model. The blue tip of the horizontal branch of the spheroid field stars is absent. The thick-disk model proposed by Gilmore and Reid does not fit the data of Kron for SA 57 and of Koo and Kron for SA 68.

Subject headings: galaxies: Milky Way — galaxies: stellar content — galaxies: structure — stars: stellar statistics

I. INTRODUCTION

The purpose of this paper is to use modern data on the distribution in color and in apparent magnitude of stars in carefully observed areas to set constraints on the large-scale distribution of galactic stars. The main results of our analysis are summarized in Table 1, which also contains a guide that shows where to find the evidence supporting each of the quantitative conclusions.

The discussion in this paper is expressed in terms of a simple model, usually referred to in the literature as the Bahcall-Soneira Galaxy model (or the B&S model). What is this model? What is assumed and what can be determined?

The geometry of the Galaxy is assumed in the B&S model to be the same as in other galaxies of the same Hubble type: an exponential disk and a de Vaucouleurs spheroid. The luminosity functions and scale heights, as well as H-R diagrams are left to be determined by local observations. The determinations are made in practice by adopting as initial values for everywhere in the Galaxy parameters that are measured for the disk and the spheroid near the Sun, supplemented by the assumption that to first approximation the field spheroid and globular cluster populations are similar. These values are used to predict the observed star colors and number counts in different directions in the Galaxy. The agreement (or disagreement) between the observed and predicted stellar data is used to delimit the acceptable functions and parameters.[1] Over the course of the past several years, we have added more detail to the model (the Wielen dip in the disk luminosity function, the ellipticity of the spheroid, and spline fits for the color-magnitude diagrams of the evolved stars) as we have matched more details in the observations.

To our initial surprise, local values for the luminosity functions and other parameters enable us to calculate with our simple geometrical model the colors and apparent magnitudes of stars that are in good agreement with the many available observations of stars in different directions and distances. The geometrical model augmented with the local luminosity functions and other parameters (specified in Tables 1, 2 and § II) will be called in this paper the "standard galaxy" model. The accuracy with which the observed and predicted color distributions and star counts agree is comparable to the uncertainties in the data (typically better than 20%).

We analyze in this paper observations in the direction of SA 57 (near the north galactic pole), of SA 68 ($b = -46°$, $l = 111°$), of SA 51 ($b = +21°$, in the direction of the anticenter), of Aquarius ($b = -57°$, $l = 36°$), and of the south galactic pole. The major sources of data are as follows: Jarvis and Tyson (1981), King (see Chiu 1980*a*), Koo and Kron (1982), Kron (1978, 1980), McLaughlin (1983), Ratnatunga (1982), Reid and Gilmore (1982), Tritton and Morton (1983), and Weistrop (1972, 1980).

We find that a single galaxy model (the standard model) is consistent with all the available data. This model is useful in summarizing a large amount of data, in delimiting the allowed range of galactic parameters, in highlighting special features of individual populations, in planning or interpreting observations for which the background density of certain types of stars is important, and as a guide to which observations are most likely to reveal new characteristics of the Galaxy.

We make many comparisons with observations. The theoretical and the observed distributions always refer to the same area of the sky as was covered in the original observations.

[1] This procedure is different from the (often unstable) classical method of analysis which involves inverting the integral equation of stellar statistics.

TABLE 1

BASIC RESULTS[a]

Subject	Result	Location
Accuracy of data..........	Δcolor $\lesssim 0.2$ mag Δ(normalization 1 bin) $\lesssim 20\%$	§§ II*c*, V; Figs. 3, 10*c*
Scale height of disk dwarfs...........	$z_s = 350 \pm 50$ pc, $5 \leq M_V \leq 13$	§ IX; Table 6; Figs. 16–17
Fluctuations in the volume density of disk dwarfs	$n/n_{\text{solar}} = 1 \pm 0.15$	§ IX; Figs. 16–17
Giant scale height	$z_s = 250 \pm 100$ pc, $M_V \leq 2.5$ mag	§ IX
Disk scale length..........	2.5 kpc $\leq h$	§ IX*b*
Wielen dip in the disk luminosity function	exists outside solar vicinity	§§ II*a*, III*a*, V; Figs. 1*a*, 4*b*, 10
Spheroid axial ratio	$b/a = 0.80^{+0.20}_{-0.05}$	§ II*d*; Table 3
Spheroid normalization ...	$n(4 \leq M_V \leq 8)$ $= 2.7(1 \pm 0.25) \times 10^{-5}$ pc^{-3}	§§ II*d*, III*b*, IV*a*, IV*b*, VI; Figs. 5, 8, 9, 12
Spheroid giants and subgiants................	exist: consistent with globular cluster color-magnitude diagrams	§§ II*b*, III*b*, IV*a*, IV*b*, V, VI*a*, VI*b*; Table 4; Figs. 2, 9*b*, 12*a*, 12*b*, 13*a*, 13*b*, 13*c*
Blue tip of the spheroid horizontal branch	absent	§ VIII*b*; Table 5
The bright end of spheroid luminosity function.................	extends to at least $M_V = +1$	§§ VII*b*, VIII*a*; Table 4; Fig. 15
Thick disk	no evidence for	§ X; Figs. 18–20

[a]See the indicated location in the text for the operational definitions of the uncertainties.

Thus, the ordinates of Figures 5–22 are stars in the magnitude (or color) interval in the observed area. *The total number of stars that were counted or predicted—in a given apparent magnitude or color interval—can be read directly from the figures*. Similarly, in our tabular comparisons we show both the measured and the calculated number of stars in the observed area. We hope that this convention will give the reader a direct impression of the uncertainties.

We obtain quantitative results for both the disk and the spheroid stars. For the disk, we obtain limits on the disk scale height and scale length, as well as on the luminosity function. For the spheroid, we determine the axis ratio, the number density at the solar position, the fraction of stars that are giants and subgiants, the bright end of the luminosity function, and a surprising characteristic of the blue tip of the horizontal branch. We also show that the "thick-disk" model proposed by Gilmore and Reid (1983) is inconsistent with the data of Kron (1978, 1980) and of Koo and Kron (1982) for faint stars in SA 57 and SA 68.

In §§ III–VII we present detailed comparisons with the data for five separate fields. We organize these sections according to field and observer. In § VIII, we analyze the bright end of the spheroid luminosity function and the blue tip of the spheroid horizontal branch. In § IX we determine the allowed range of the disk scale heights and of fluctuations in the volume density; we also set a lower limit on the disk scale length. In § X we present calculations based upon the thick-disk model of Gilmore and Reid. We summarize our conclusions in § XI.

In Appendix A, we investigate whether the feature labeled "GCF" in Figure 1*a*—which is observed in globular cluster luminosity functions—is also present in the luminosity function of field spheroid stars. In Appendix B, we discuss the density distribution of K giants perpendicular to the galactic plane.

The reader is urged to read first § XI, the summary and discussion section, in order to get an overview of the paper.

II. THE GALAXY MODEL

a) *Tactics and Basic Ingredients*

The basic departure we make from the traditional work on star counts is that we *assume* the geometric shape of the stellar components, using the photometric observations of other galaxies as an indication of the form of the density laws of the Galaxy. In previous work, star counts were used to try to establish the general structure of the stellar distribution. In our opinion, the overall shape of the Galaxy is best determined by analogy with other galaxies, exploiting the measurements of star counts and colors in the Galaxy to fix scale parameters, density normalization factors, and luminosity functions. As a *tactical* approach, we avoid areas where the obscuration is appreciable. In the early work, star counts were used to try to measure the galactic obscuration. (For a description of the traditional approach, see, e.g., Seares 1924; Bok 1937; Oort 1938, and references therein; as well as the more recent work of the Basel group as described in Becker 1965; Becker and Steppe 1977; and the Edinburgh group as described in Gilmore and Reid 1983).

The stellar density laws used for the disk and spheroid in our standard model of the Galaxy are given in Table 2. We use an exponential disk for the Population I stars and a de Vaucouleurs (1959) law for the Population II spheroid stars. For the discussion in this paper, only the disk and the spheroid are required. We have tested the sensitivity of our inferences to the assumed geometry by performing (see Bahcall and Soneira 1980*a*, hereafter Paper I) a series of calculations with varying model parameters. We also investigated exponential disks with holes around the galactic center and spheroid mass distributions that are described by a Hubble (1930) instead of a de Vaucouleurs law. The variations in geometry, about the standard assumptions, that were considered in Paper I did not produce differences in the predicted counts that were large enough to be detected easily in the available data.

The adopted luminosity function of the disk stars, which is shown in Figure 1*a*, was determined by Wielen (1974) from observations of nearby stars brighter than $M_V = 12.5$. We assumed for definiteness that the disk luminosity function is constant between absolute visual magnitudes of 12.5 and 16.5 (i.e., down to the end of hydrogen-burning main-sequence stars) and is equal to the observed value at $M_V = 12.5$. (The data considered in this paper do not set useful constraints on the disk luminosity function in the region in which it is assumed flat.) The total number of stars brighter than $M_V = 16.5$ is then 0.13 stars pc^{-3}. In Paper I, we used a slightly different analytic approximation for the disk luminosity function; the comparison between the predicted counts obtained using the analytic and the Wielen luminosity functions is made in §§ III and V and in Armandroff (1982).

There are many determinations in the literature of the local scale heights of disk stars (see references in the Fig. 2 legend of Paper I). In accordance with this information, the older main-sequence stars (with $M_V > +5.1$) were assumed to have an exponential scale height of 325 pc, and the younger main-sequence stars (with $M_V < +2.3$) were taken to have a scale height of 90 pc. The scale heights are denoted by $H(M_V)$ in Table 2. For M_V between +2.3 and +5.1, the scale height was linearly interpolated between 90 and 325 pc (see Fig. 2 of Paper I). We assume that the disk giants have an average scale height of 250 pc and that the white dwarfs have the same scale height as the old stars. The observational constraints on the

TABLE 2

ASSUMED STELLAR DISTRIBUTIONS

Component	Distribution
Disk	$n_D = n_D(R_0)\exp[-z/H(M_V)]\exp[-(x-R_0)/h]$ $\times 1.25(R/R_0)^{-6/8}\left\{\exp\left[-10.093(R/R_0)^{1/4}+10.093\right]\right\}, \quad R < 0.03R_0$
Spheroid	$n_{\rm sph} = n_{\rm sph}(R_0)(R/R_0)^{-7/8}\left\{\exp\left[-10.093(R/R_0)^{1/4}+10.093\right]\right\}$ $\times\left[1-0.08669/(R/R_0)^{1/4}\right], \quad R \geq 0.03R_0$
Normalization ...	$n_D(R_0) = 0.13$ pc^{-3}, $n_{\rm sph} = 0.00026$ pc^{-3}, for $M_V \leq 16.5$ mag

NOTE.—Here z is the distance perpendicular to the plane, x is the galactocentric distance in the plane, and h is the disk scale length. Galactocentric distance $R = (x^2 + z^2/\kappa^2)^{1/2}$, where κ is the axis ratio and $1-\kappa$ is the ellipticity. We adopt $R_0 = 8$ kpc and $h = 3.5$ kpc. This analytic form of the de Vaucouleurs law is taken from Young 1976. The fraction of disk stars that are on the main sequence is given, in the plane of the disk, by eq. (1) of Paper III.

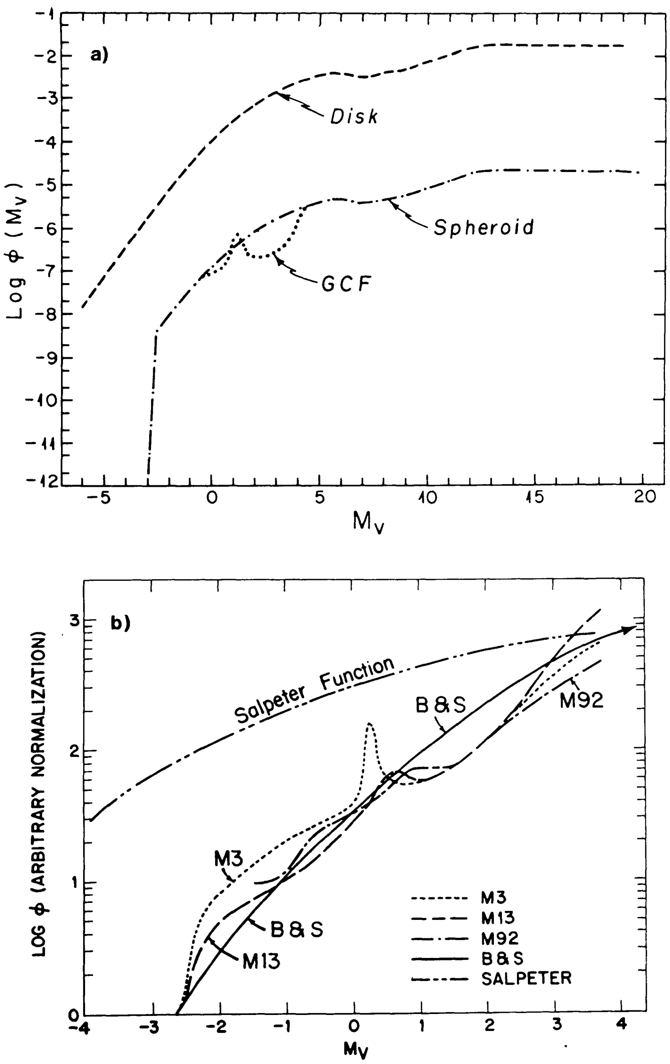


FIG. 1.—(a) Adopted disk and spheroid luminosity functions. The luminosity function for the disk is taken from Wielen (1974) for $M_V < 12.5$ and is assumed constant for $M_V > 12.5$. For $M_V \gtrsim +3$, the spheroid luminosity function is determined observationally by faint star counts ($+3 \lesssim M_V \lesssim +8$; see Papers I and IV) and by proper-motion studies of high velocity stars ($+6 \lesssim M_V \lesssim +11$; see Schmidt 1975). For simplicity, we have adopted the Wielen function also for the spheroid; this luminosity function is equivalent, for magnitudes brighter than $M_V = 6$, to the analytic curve used in Paper I. The feature labeled "GCF" is included in calculations that are described in Appendix A. (b) Comparisons with spheroid luminosity function. Our approximate spheroid luminosity function is shown as the continuous curve labeled B&S. The measured luminosity functions for M3 (Sandage 1954), M13 (Simoda and Kimura 1968), and M92 (Tayler 1954) are shown for comparison. The Salpeter (1955) unevolved luminosity function is also given.

scale heights are determined in § IX for the data considered in the present paper.

The scale length of the spheroid that is implied by the density law given in Table 2 was adopted following de Vaucouleurs (1977) and de Vaucouleurs and Buta (1978). We use a value of 8 kpc for the distance of the Sun from the center of the Galaxy. The results we have obtained so far are not sensitive to the precise values of the solar position or the linear scale factors for the spheroid because the density functions are normalized in the solar neighborhood.

On the other hand, the results in Paper I and in Bahcall, Schmidt, and Soneira (1983; hereafter Paper IV), *are* sensitive to the normalization at the Sun (in stars per pc^{-3}) of the spheroid density. For relatively bright stars, e.g., $m_V < 19$, the predicted color distributions and star counts are also sensitive to the color-magnitude diagram of the giant branch of the spheroid stars (see Bahcall *et al.* 1983, hereafter Paper VI). Therefore, we can use star counts as a function of color and apparent magnitude to fix the spheroid normalization and to set constraints on the color-magnitude diagram of the spheroid.

In a magnitude-limited survey, the predicted star counts are sensitive to a limited range of the spheroid luminosity function. In Paper IV, star counts in two colors were used to determine the spheroid luminosity function between about $M_V = +3$ and $+8$.

The luminosity function for the spheroid that we have adopted as a first approximation is compared in Figure 1a with the disk luminosity function and, in Figure 1b, with the measured luminosity functions of the globular clusters M3, M13, M92, and the Salpeter (1955) unevolved luminosity function. We have included the Wielen (1974) dip in the spheroid luminosity function (see Fig. 1a near $M_V = 7$) as our standard option, although we make calculations in later sections with and without the dip (see especially the discussion in §§ IIIa and V, Figs. 4b and 10). In previous papers, we used a slightly different formula for the spheroid luminosity function which is given by equation (1) of Paper I. We have also included a cutoff in the spheroid luminosity function at $M_V = -3.0$ mag, since no globular cluster stars are observed brighter than this limit.[2] The feature labeled "GCF"—globular cluster feature—is not included in our standard model. We conclude in Appendix A that the available data are not sufficient to determine if this feature is present in the luminosity function of the spheroid field stars, although it is present in the luminosity functions of several globular clusters (see, e.g., Da Costa 1982). Both the old and the new spheroid luminosity functions resemble the measured luminosity functions of globular clusters in the Galaxy within the rather large scatter among the individual functions and are consistent with the Schmidt (1975) data on high velocity stars (sensitive primarily to stars with $+6 < M_V < +12$). In previous studies (see especially Papers I and IV) we have shown that the assumed analytic approximation to the spheroid luminosity function is a satisfactory fit to the data on faint star counts (sensitive primarily to stars with $+3 < M_V < +8$). One of the aims of the present work is to determine if the available observations on brighter stars provide evidence that the spheroid luminosity function is different from the smooth function shown in Figure 1a.

The color-magnitude diagram of the spheroid is important for observations in two or more colors that refer to inter-

[2] None of the calculations or comparisons with data that are discussed in the present paper are affected by this assumption. The number of predicted spheroid stars with $M_V < -3.0$ mag would be too small to show on any of our diagrams even if we assumed that the luminosity function extends smoothly down to $M_V = -6.0$ mag.

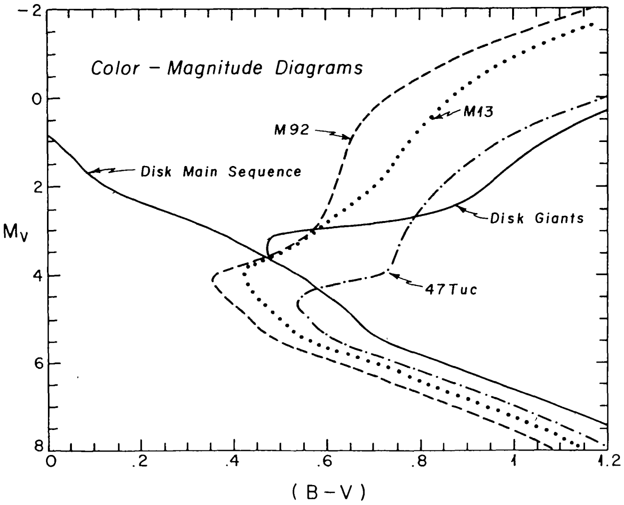


FIG. 2.—Color-magnitude diagrams. Color-magnitude diagrams for M13, M92, and 47 Tuc are taken from Sandage (1970, 1982), Hesser and Hartwick (1977), and Lee (1977). The disk color-magnitude diagram is shown for comparison. The disk diagram was constructed from main-sequence data given by Johnson (1965, 1966) and Keenan (1963), with the giant branch assumed to be the same as given by Morgan and Eggleton (1978) for M67.

mediate apparent magnitudes (e.g., B and $V<19$). We show in Figure 2 the color-magnitude diagrams that we have used for illustrative purposes in the present paper. The correct spheroid color-magnitude diagram probably has contributions from a number of different components or metallicities (see, e.g., the excellent review by Kraft 1983). We have chosen the color-magnitude diagrams of the globular clusters M92 and 47 Tuc as examples of the extreme range of known Population II color-magnitude diagrams with very different metallicities; they have, respectively, $[Fe/H] \approx -2.1$ and ≈ -0.6 (see Harris and Racine 1979; Zinn 1980). We also use the color-magnitude diagram of M13, with $[Fe/H] \approx -1.4$ (Harris and Racine 1979), since it provides a good fit to much of the high latitude data.

b) *Previous Comparisons with Observations*

The model that is described above fits well (Paper I) the frequency-color diagrams for SA 57 and SA 68 that were reported by Kron (1978, 1980). For these two fields and in the magnitude range that Kron's data include ($19.75 \lesssim m_V \lesssim 22.0$), the disk and the spheroid appear as two separate peaks in the frequency of occurrence in both the predicted and the observed counts. The predicted frequency-color diagrams are different in some other directions and apparent magnitude ranges. The results described above were obtained in Paper I with a model spheroid that had an axial ratio of unity and a color-magnitude diagram like M67. We improve this analysis in the present paper (§§ III*b*, IV*a*) using the new data of Koo and Kron (1982), a spheroid that departs from round and has a spheroid color-magnitude diagram like some representative globular clusters.

The calculated (Fig. 4*a* of Paper I) and observed differential star counts averaged over galactic longitude agree with the available star counts for galactic latitudes, $|b| \geq 20°$, sufficiently large that obscuration is not important.

The model fits well (see Paper VI) the distribution in $B-V$ of stellar colors observed by Tritton and Morton (1982, 1983) for stars with $m_B \leq 19$ in a field in the direction of Aquarius ($b=-51°$ and $l=36.5°$) *provided* an appropriate color-magnitude diagram is used to calculate the colors for the spheroid stars. Tritton and Morton (1982) showed earlier that their observations were not fitted by the B&S model if they assumed that all the spheroid stars were on the main sequence. We show in the present paper that the absolute density normalizations are also given correctly for the Aquarius region if one assumes the oblate spheroid mass distribution that is determined from an analysis of SA 57 and SA 68.

In the present work, we improve the analysis in several ways, including taking account of the probable oblateness of the spheroid, using the local Wielen (1974) luminosity function, performing calculations with spheroid color-magnitude diagrams like representative globular cluster diagrams, using improved and additional data, and considering two new fields (SA 51 and the south galactic pole). Also, we determine the allowed range for a number of characteristic parameters of the Galaxy.

c) *Accuracy of Data*

How accurate are the available data? Systematic uncertainties are expected to be large compared to statistical errors, especially for the colors. Examples of possible systematic uncertainties include errors in calibration magnitudes, uncertainties in the wavelength sensitivity of the bands used, incompleteness of the sample, and contamination by galaxies. There are well-known examples in the literature in which significant systematic errors have been found in apparent magnitude scales and in color-differences of stars within the magnitude range in which we are interested (see, e.g., Stebbins, Whitford, and Johnson 1950; Faber *et al.* 1976). In order to be able to decide which theoretical models fit the data satisfactorily, we must make some estimate of the likely uncertainties in the existing data.

The best way to estimate the systematic uncertainties is to compare the results obtained by different observers using independent techniques. Unfortunately, there are few fields in which the same observational parameters have been determined by different observers. We discuss below three illustrative cases for which multiple observations are available. The actual systematic uncertainties may be larger than are implied by the comparisons made below. Some systematic uncertainties are not operative in the cases we discuss because the data being compared are not independent.

Figure 3*a* compares the star counts in SA 57 that we derived from the data of Weistrop (1972, 1980) using two different magnitude systems. The counts were reduced in two ways: (1) using the apparent magnitude scale and colors of Weistrop (1972); and (2) using the same star counts with the recalibrated apparent magnitude scales and colors of Faber *et al.* (1976). In both cases, we have determined the number of stars of different colors directly from a magnetic tape generously supplied to us by Weistrop (1980). The magnitude range shown in Figure 3*a* is $m_V=12$–16, the region in which the

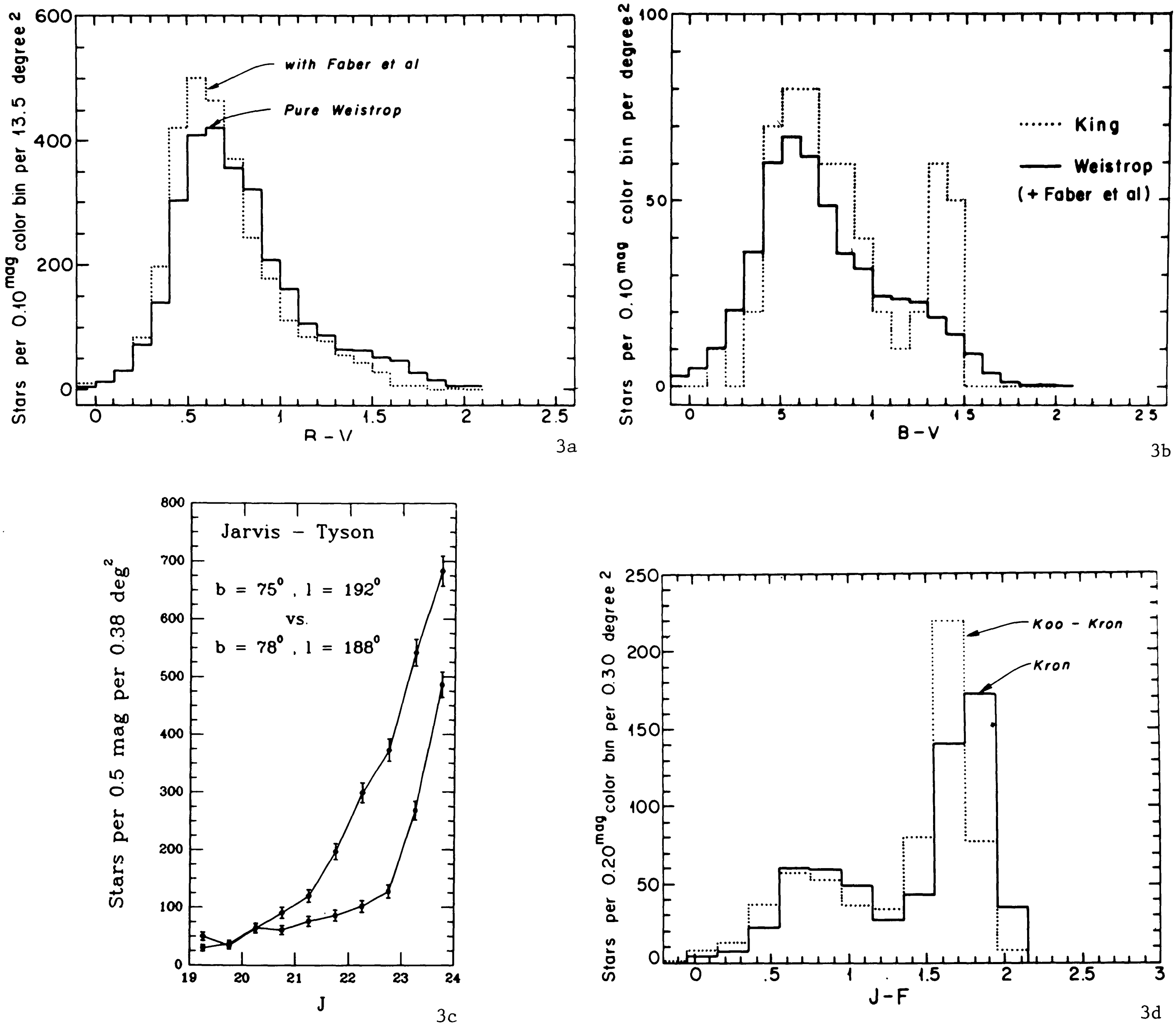


FIG. 3.—Accuracy of the data. (*a*) Comparison of colors of bright stars in SA 57. Color histograms obtained in SA 57 using the data of Weistrop (1972) and the magnitude systems of Weistrop (1972) and of Faber *et al.* (1976) are shown. Histograms refer to 2.7×10^3 stars between $m_V=12$ and 16 mag. (*b*) Comparison of the observations of Weistrop (1972) to $V=18$ with the observations of King (see Chiu 1980*a*) for SA 57. (*c*) Comparison of the differential star counts in two-adjacent high latitude fields studied by Jarvis and Tyson (1981). Error bars were calculated from Poisson statistics. (*d*) Comparison of colors of faint stars in SA 68. Color histograms obtained in SA 68 using the data of Kron (1978, 1980) and the improved data of Koo and Kron (1982). Histograms refer to 620 stars between $(J+F)/2=19.95$ and 22.15 mag.

data are expected to be of relatively high quality for *both* photometric systems. Figure 3*a* shows color shifts of features of order 0.1 mag and differences in the number of stars within a given color bin of order 25%.

Figure 3*b* compares the Weistrop data with data from King (see Chiu 1980*a*). Note that there is a feature in King's data (near $B-V=1.4$ mag) that is not present in Weistrop's star counts; this peak contains only 11 stars in the original data. There are a total of 58 stars in King's sample to $V=18$ and 6900 in Weistrop's sample to the same magnitude limit.

We conclude that, even at the bright apparent magnitudes considered here and in the standard $B-V$ color band, significant uncertainties may exist in the available data.

Figure 3*c* compares the star counts reported by Jarvis and Tyson (1981) for two high latitude fields separated by only a few degrees. The differential star counts in the adjacent fields differ by typically a factor of 2. The expected difference—according to the standard galaxy model—is only 2%. Similar discrepancies exist between the differential star counts reported by Jarvis and Tyson for other high galactic latitude fields (see their Table 8).[3]

Figure 3*d* compares the color historgram in SA 68 that describes the star counts in J and F obtained by Kron (1978, 1980) and by Koo and Kron (1982); the later results were derived using improved reduction techniques. The counts include all stars between $(J+F)/2=19.95$ and 22.15 in 0.3 deg^2 centered on SA 68. We are grateful to D. Koo and R. Kron for providing us with their important results in a convenient form. We see from Figure 3*d* that shifts of order 0.2 mag in the color bins at which certain features occur are possible and the number of stars in a given color bin may change by as much as 50%. In the Kron versus Koo-Kron comparison, much of the apparent change is caused by the finite bin size.

The comparisons made above suggest that rather large differences are necessary—with the available data—to distinguish between theoretical models computed with different parameters or assumptions. As a rough guide, we will accept shifts of up to 0.2 mag in color of certain features and of order 25% in the number of stars in a given color bin. Based upon the comparison of total star counts made by different observers in the direction of the north galactic pole (see Figs. 4*c* and 4*d*), we will require agreement with the total number counts to an accuracy of better than 20%. In order to increase the margin of safety without greatly decreasing the information content of the observations, we shall compare our calculations with color distributions and star counts to 1 mag brighter than the quoted observational limits.

d) *Axis Ratio and Normalization of the Spheroid*

The axis ratio of the spheroidal star distribution can be determined (Papers I and II) by comparing the number of spheroid stars that are observed in different fields that lie in the plane perpendicular to the vector directed from the Sun to the galactic center (i.e., in the $l=90°$, $270°$ plane). The number of spheroid stars with a given magnitude or color will be independent of galactic latitude in this plane if the spheroid is perfectly round (neglecting obscuration, which is valid for the case that is considered below). In Paper I, the star counts by Kron (1978, 1980) in SA 57 and in SA 68 were used to estimate, assuming the spheroid had a color-magnitude diagram like M67, an axial ratio of 0.85. We repeat the analysis here using the color-magnitude diagrams of M13, M92, and 47 Tuc, and the estimates described above of the possible sources of uncertainties.

Spheroid stars can be separated, in the Kron data, from disk stars in a frequency-color diagram. Stars bluer than $J-F=1.35$ mag are nearly all spheroid stars (see Figs. 5*a* and 8*c* of §§ III and IV). The ratio of the number of such blue stars in SA 57 to SA 68 is, in the Kron data, 1.09. Taking into account the possibility of color shifts as large as 0.2 mag, we find a ratio of $1.09^{+0.14}_{-0.17}$ for the number of spheroid stars in the direction of SA 57 to the number of spheroid stars in the direction of SA 68.

Table 3 shows the computed ratio of spheroid stars in SA 57 to the number in SA 68 for a variety of assumptions.[4] The best fit to the Kron data corresponds to an axis ratio of 0.80, with the observational uncertainty corresponding to an axis ratio of between 0.75 and 1.0. We list in Table 3 the calculated ratios that were obtained using spheroid luminosity functions both with and without the dip found by Wielen (1974) in the luminosity function near $M_V=7$ (see Fig. 1*a*), and we also tabulate the calculated ratios for all colors bluer than $J-F=1.15$, 1.35, and 1.55. The numbers in Table 3 were calculated using best-fit color-magnitude diagrams corresponding to M13 for SA 57 and 47 Tuc for SA 68 (see § III*a*). The calculated ratios are not sensitive to the choice of these particular color-magnitude diagrams.

In summary, we conclude that the axis ratio, b/a, of the spheroid stars satisfies, with the Kron data set and the assumed uncertainties,

$$\frac{b}{a}=0.80^{+0.20}_{-0.05}. \tag{1}$$

This result is in good agreement with the determination by Oort and Plaut (1975), who found $0.8\leq b/a\leq 1.0$ for RR Lyrae variables in fields near the galactic center. Fall (1981) measured the axial ratio at galactocentric radii of ~ 2 kpc and found $b/a\approx 0.8$ (see Fig. 1 of Fall 1981 for the beautiful picture of the galactic bulge taken by R. E. Royer). The result we obtain from star counts, equation (1) above, refers to galactocentric distances of ~ 10 kpc, whereas the Oort and Plaut and the Fall determinations refer to much smaller distances. We conclude that there is no evidence for a large gradient in the axial ratio of the spheroid stars between 2 and 10 kpc. Frenk and White (1982) have reached similar conclusions for the globular cluster system, for which they find that $b/a\approx 0.85\pm 0.13$ (see also the earlier discussions referenced in the Frenk and White paper). Boroson (1981) finds that for eight spiral galaxies whose values of b/a could be determined reliably that $0.6\leq b/a\leq 0.9$.

In what follows, we shall adopt an axis ratio of 0.80 for our standard models.

The number of spheroid stars between $M_V=4$ and 8 at the solar position can be calculated from the best fit to the Kron data in either SA 57 or SA 68. We find, using the form of the spheroid density law given in Table 2,

$$n(4\text{ mag}\leq M_V\leq 8\text{ mag})=2.65\times 10^{-5}\text{ pc}^{-3}. \tag{2}$$

The uncertainty in this normalization is at least as large as 25% with the available data. The total number of spheroid stars at the solar position can be estimated by integrating the

[3] Koo and Kron (1982) have carried out an informative comparison of their work on SA 68 with that of Jarvis and Tyson (1982). Koo and Kron suggest that the Jarvis-Tyson analysis does not properly distinguish between stars and galaxies at faint magnitudes.

[4] We have carried out calculations for SA 68 using the 0.1 mag of obscuration estimated by Chiu (1980*b*) and the 0.02 mag implied by the Sandage (1970) model, as well as an upper limit estimate of 0.2 mag. The uncertainty introduced by the obscuration is small compared to the other uncertainties we describe explicitly.

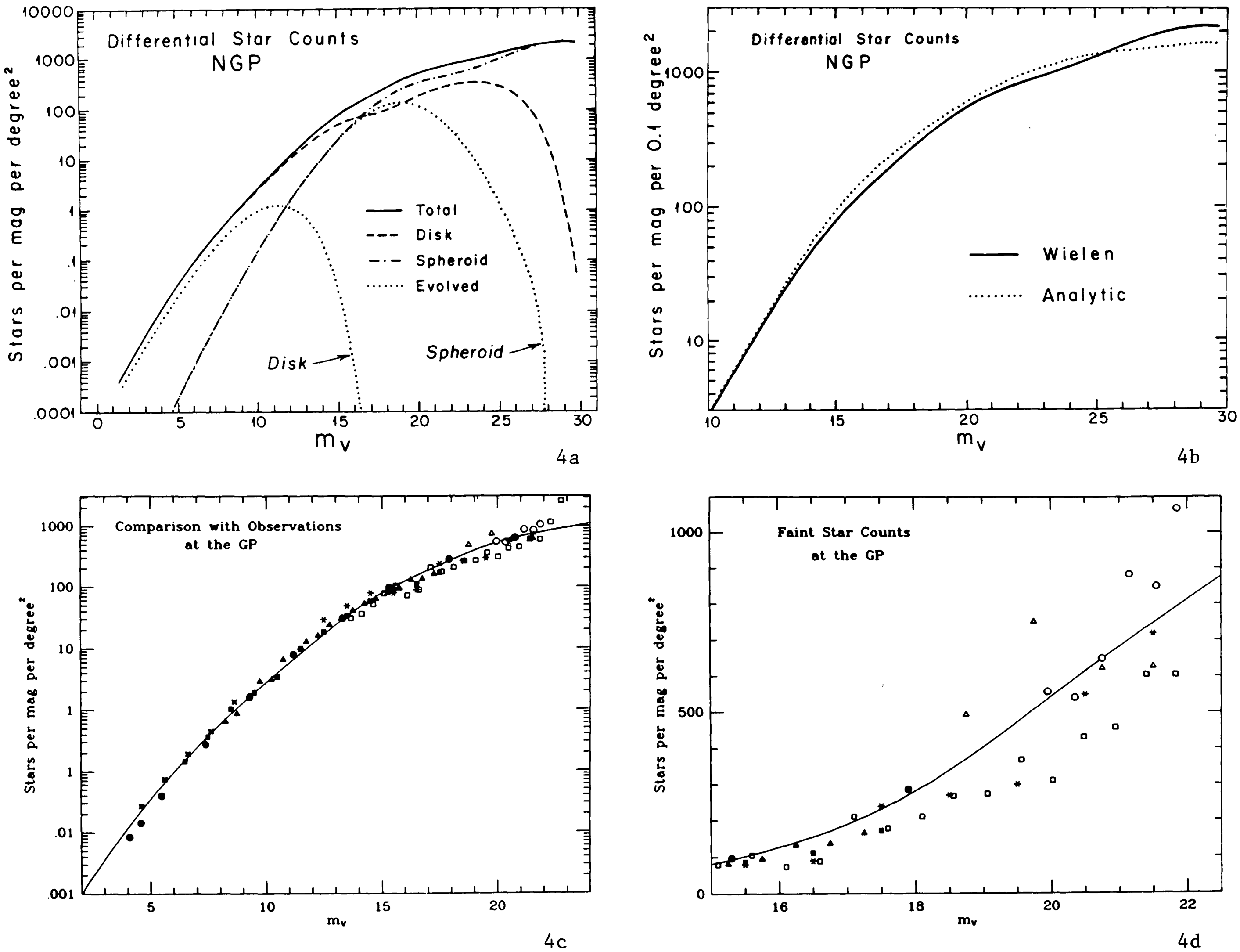


FIG. 4.—North galactic pole. (a) Relative contributions to the star counts. Predicted differential star counts are shown as a function of visual apparent magnitude. The standard B&S model, described in § II, was used for the calculations. Individual contributions of the disk and the spheroid stars, each with their associated giant and subgiant components, are also shown. (b) Comparison of the star counts predicted by the Wielen and the analytic luminosity functions. Differential star counts calculated using the Wielen (1974) luminosity function and the analytic luminosity function given in eq. (1) of Paper I are compared. The maximum difference occurs at $m_V \approx 16.5$ and is $\sim 20\%$. (c) Differential star counts mag^{-1} deg^{-2} for the galactic pole. Solid curve is predicted by the standard model. Data from Seares *et al.* (1925) as reduced to the visual band in Paper I are plotted as filled circles, data from Weistrop (1972, 1980) with Faber *et al.* (1976) corrections as filled squares, data from McLaughlin (1983) as open crosses, data from Reid and Gilmore (1983) as filled triangles, data from King (Chiu 1980a) as asterisks, data from Jarvis and Tyson (1981) as open squares, data from Peterson *et al.* (1979) as open triangles, data from Kron (1978, 1980) as open circles. (d) Same as (c), but with a linear scale.

luminosity function shown in Figure 1a using the density normalization given in equation (2). We find

$$n(M_V \leq 16.5 \text{ mag}) \approx 3 \times 10^{-4} \text{ pc}^{-3}. \quad (3)$$

Stars outside the observationally accessible range could contribute a larger number density (see, e.g., Paper IV, especially § IV, for some examples of models in which this uncertainty is evaluated). The total mass and luminosity of the spheroid have been estimated in Paper IV.

The spheroid normalization given in equations (2) and (3) is $\sim 1/500$ times the number density of disk stars locally.

III. NORTH GALACTIC POLE

a) General Results

The differential star counts for the galactic pole that are predicted by the standard model, which is described in § II, are shown in Figure 4a for apparent visual magnitudes between 2 and 30. The contributions to the counts from the disk and the spheroid are shown separately in Figure 4a; the contributions from the subgiants and giants in each component are indicated by dotted lines. The disk dominates the calculated star counts brighter than $m_V = 15$, and the spheroid

TABLE 3

CALCULATED RATIO OF SPHEROID STARS IN SA 57 AND SA 68 AS A FUNCTION OF THE SPHEROID AXIS RATIO[a]

Axis Ratio	Reddest Color for Spheroid Stars	Stars: SA 57/SA 68 (with Wielen LF)[b]	Stars: SA 57/SA 68 (without Wielen dip)
1.0	1.15	1.48	1.54
1.0	1.35	1.45	1.49
1.0	1.55	1.24	1.36
0.85	1.15	1.15	1.20
0.85	1.35	1.14	1.19
0.85	1.55	1.00	1.10
0.80	1.15	1.06	1.16
0.80	1.35	1.06	1.10
0.80	1.55	0.94	1.00
0.75	1.15	0.97	1.01
0.75	1.35	0.97	1.02
0.75	1.55	0.88	0.94
0.70	1.15	0.85	0.81
0.70	1.35	0.87	0.92
0.70	1.55	0.83	0.88
0.60	1.15	0.74	0.78
0.60	1.35	0.75	0.80
0.60	1.55	0.65	0.70
0.50	1.15	0.61	0.65
0.50	1.35	0.65	0.68
0.50	1.55	0.64	0.64

[a]Magnitude range is $(J+F)/2 = 19.95$ to 22.15 mag.
[b]See Fig. 1*a*.

is most important for magnitudes fainter than $m_V = 18$. Most of the spheroid stars are predicted to be giants and subgiants brighter than about $m_V = 18$. The disk giants are expected to be important only at the brightest apparent magnitudes, brighter than about $m_V = 9$.

Upgren and Armandroff (1981) have recently stressed the significance of the difference between the disk luminosity function determined by Luyten (1968) using his extensive proper-motion data and the luminosity function obtained by Wielen (1975) from the Gliese (1969) catalog. The difference between the two luminosity functions is shown clearly in Figure 1 of the paper by Upgren and Armandroff, which also contains a discussion of the previous indications that the two luminosity functions are significantly different. A careful reevaluation of the completeness of the available stellar samples by Upgren and Armandroff supports the lower value for the disk luminosity function given by Wielen in the range of absolute visual magnitudes between +6 and +9.

Figure 4*b* compares the predicted differential star counts that were computed using the Wielen (1974) luminosity function and the analytic function, which represents the Luyten (1969) luminosity function, that was given in Paper I (eq. [1]). The two functions were normalized to the same value for the brightest stars. The maximum difference caused by the Wielen dip in the disk luminosity function occurs at $m_V \approx 16.5$, where it amounts to ~ 20%. The maximum difference caused by the dip in the spheroid luminosity function appears at $m_V \approx 23$ and is ~15%.

Figures 4*c* and 4*d* compare all of the available data at the north galactic pole with the predicted differential star counts. The data of Reid and Gilmore (1982) at the south galactic pole are also included for completeness. The observed star densities vary by five orders of magnitude from $m_V = 4$ to 22 mag; this dependence is well represented by the predicted star counts. Figure 4*d* compares on an expanded scale the predicted and observed star counts in the region between $m_V = 15$ and 22 mag. The predicted curve lies above most of the Jarvis and Tyson (1981) data, but by an amount ($\lesssim 30\%$) that is less than the internal disagreement between the Jarvis-Tyson data in two high latitude fields (see Fig. 3*c*) and between the Jarvis-Tyson data and some of the other data in Figure 4*d*.

We conclude that the predicted differential star counts are in agreement with the observed star counts to an accuracy comparable to the uncertainties in the data, throughout the five orders of magnitude in star density over which the observed counts vary.

b) Comparison with Kron's Data: $(J+F)/2 = 19.95$ to 22.15

Figure 5*a* compares the color distribution predicted by the standard galaxy model with the star colors observed by Kron (1978, 1980) for faint stars with $(J+F)/2$ magnitudes between 19.95 and 22.15. The total number of stars in the observed sample is 482. The field is centered on SA 57: $b = 86°$, $l = 65°$. To the accuracy with which we work, this may be regarded as the north galactic pole.

The ordinate of Figure 5*a* shows—as do all similar comparisons in this paper—the number of stars that are expected in a sample covering the same angular area as the observed sample. The number of observed stars per bin can be read directly from the figure. The color distribution that is shown was obtained by convolving the colors obtained from the model with a Gaussian color dispersion having a standard deviation $\sigma_{\rm color} = 0.1$ mag. This dispersion represents both the measuring errors and the intrinsic dispersion in color within the color-magnitude diagram of the spheroid (see Paper VI). We have performed a similar convolution for all other color histograms representing the B&S model that are shown in this paper. Except where we state explicitly otherwise, we have always assumed a color dispersion of 0.1 mag.

The contributions from the disk and spheroid are indicated separately in Figure 5*a*. The number of spheroid stars that are expected to be giants and subgiants—labeled "evolved" in the figure—is also shown; the color-magnitude diagram of M13 (see Fig. 2) was used for this calculation. The number of disk giants and subgiants is expected to be effectively zero for the magnitude range under discussion.

The spheroid stars are clearly separated from the disk stars in this magnitude range and direction, a fact that was used in the determination of the spheroid eccentricity in § II*d*. About one-fourth of the stars in the spheroid peak are expected to be stars that have already evolved off the main sequence.

Figure 5*b* compares the observed and calculated star distributions, now both binned in color intervals of 0.2 mag, as in the data. It is instructive to compare Figure 5*b* with Figure

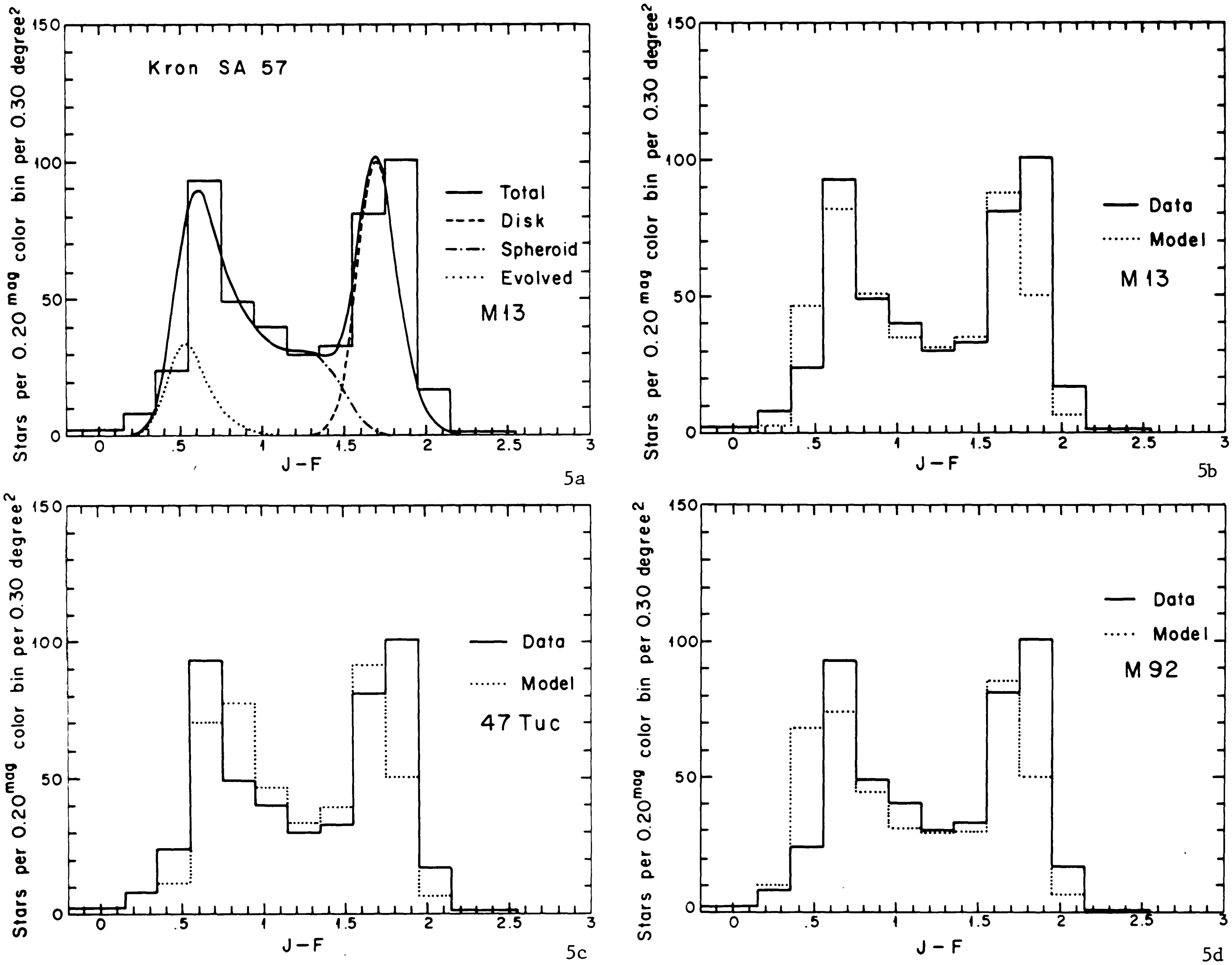


FIG. 5.—Comparison of calculated color distributions with Kron's observations of faint stars in SA 57. Kron's sample contains stars between $(J+F)/2 = 19.95$ and 22.15 mag. The ordinate gives, both for the observations and the calculations, the number of stars in 0.3 deg^2. Calculated colors have been convolved with a color dispersion having a standard deviation of 0.1 mag in $J-F$. The color-magnitude diagram of M13 was used for the spheroid in calculating the distributions shown in 5(a) and (b). Color-magnitude diagrams of 47 Tuc and M92 were assumed for the spheroid in the calculations shown, respectively, in (c) and (d). The contribution of subgiants and giants is denoted by the curve labeled "evolved."

3d, which shows the histograms of the Kron and of the Koo-Kron data for SA 68.

The color-magnitude diagram of 47 Tuc (see Fig. 2) was used to calculate the model distribution of colors that is also compared with Kron's data in Figure 5c. The color-magnitude diagram of this metal-rich globular cluster predicts a somewhat redder than observed distribution of $J-F$ colors. Figure 5d shows the color histogram that was calculated using a spheroid model with a color-magnitude diagram like the metal-poor globular cluster, M92 (see Fig. 2). In this case, the model predicts a somewhat bluer than observed color distribution.

We conclude that—for the assumed luminosity function (Fig. 1a)—the Kron data are marginally more consistent with an intermediate metallicity spheroid component (like M13) at the north galactic pole than with a color-magnitude diagram describing either the metal-rich globular cluster, 47 Tuc, or the metal-poor globular cluster, M92.

c) Comparison with Weistrop's Data: $V \leq 16$

We have counted stars on a data tape kindly supplied by D. Weistrop, which was acquired in her important study of 13.5 deg^2 at the north galactic pole (see Weistrop 1972). The photometric standards of Faber *et al.* (1976) were used in converting Weistrop's magnitudes and colors to the magnitudes and colors shown in Figure 6. We have adopted equation (6) of Faber *et al.* and have distributed the color error equally between B and V (King 1980). This sample contains the largest number of stars at the north galactic pole (~ 2900

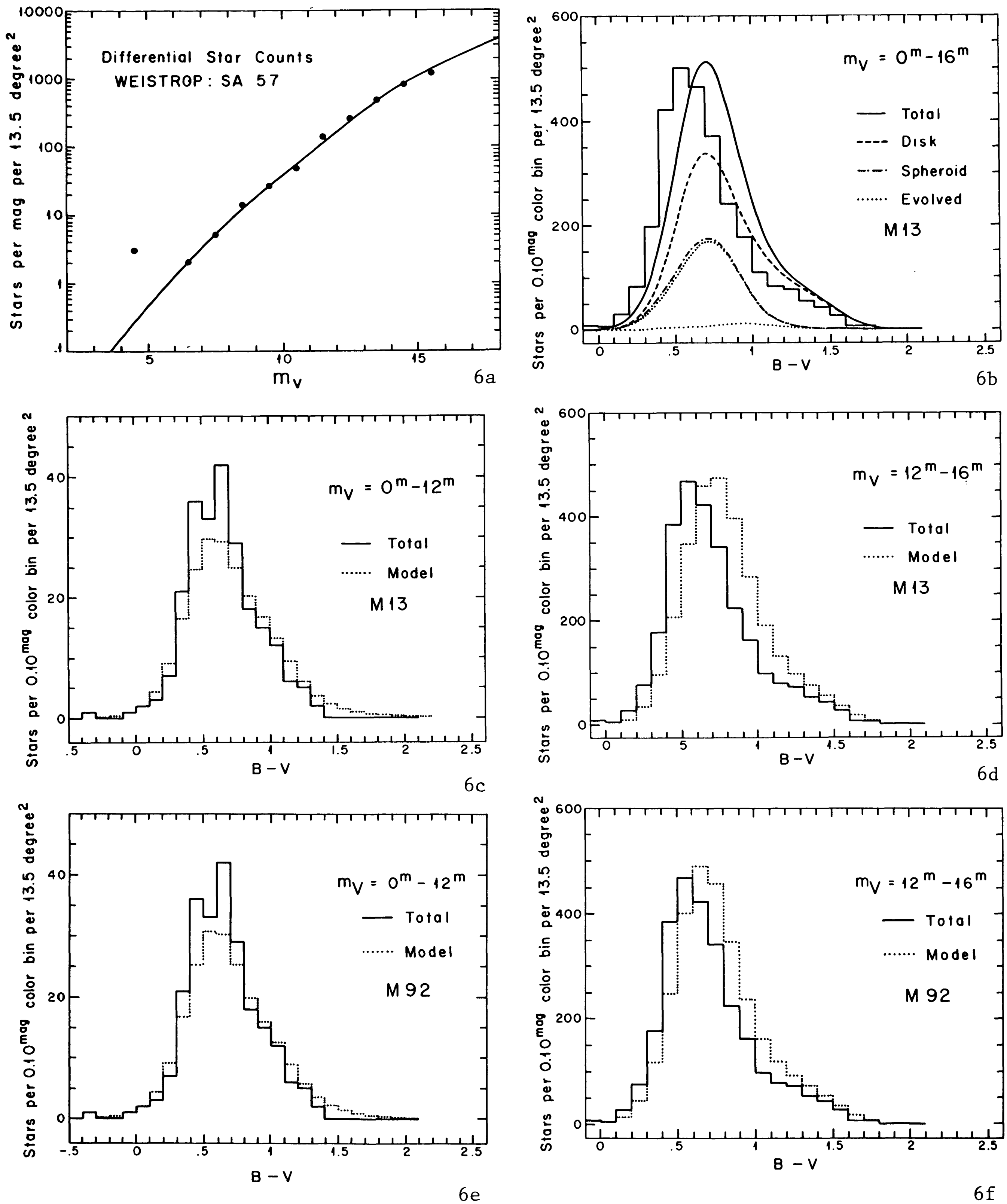


FIG. 6.—Comparison with star counts and color distributions by Weistrop of bright stars in SA 57. (*a*) Calculated and observed differential star counts. (*b*)–(*d*) Comparison of the observed color histogram with the histogram predicted by the B&S model assuming a spheroid color-magnitude diagram similar to that of M13; (*e*)–(*f*) make a similar comparison using an M92 spheroid color-magnitude diagram. The magnitude range for the various panels is (*a*) $m_V = 0$ to 16; (*b*) $m_V = 0$ to 16; (*c*) $m_V = 0$ to 12; (*d*) $m_V = 12$ to 16; (*e*) $m_V = 0$ to 12; and (*f*) $m_V = 12$ to 16. Following the advice of Weistrop (1980), we have used a color dispersion of $\sigma_{B-V} = 0.15$ mag for all of the figures. The contribution of subgiants and giants is denoted by the curve labeled "evolved."

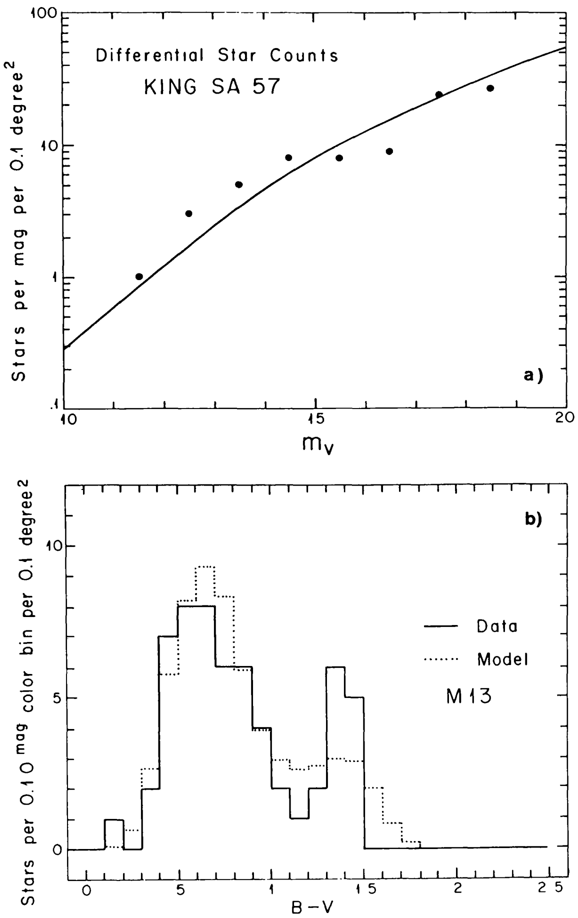


FIG. 7.—Comparison with King's star counts and color distributions in SA 57. (*a*) Comparison of Data of I. King (as published in Chiu 1980*a*) and the differential star counts predicted by the standard Galaxy model; (*b*) compares the color distributions assuming a spheroid color-magnitude diagram similar to that of M13. The magnitude limit in both figures is $m_V = 18$.

to $V = 16$), although the corrected photometry of Faber *et al.* can only be applied reliably to relatively bright magnitudes, $m_V = 16$. For a discussion of some uncertainties in these data, see § II*c* and Figures 3*a*–3*c*.

The differential star counts versus visual apparent magnitude are shown in Figure 6*a* for the Weistrop data. The calculated and observed results are in good agreement. The color distributions calculated using different color-magnitude diagrams for the spheroid are compared in Figures 6*b*–6*f* with the observed distribution. The calculated and observed distributions are in excellent agreement at bright magnitudes, for a spheroid with either an M13 or an M92 color-magnitude diagram. At the fainter magnitudes, the M92 color-magnitude diagram gives a slightly better fit.

d) *Comparison with King's Data: $V \leq 18$*

I. King has obtained data in *B* and *V* for a small field, 0.1 deg^2, near the north galactic pole. Although there are only 58 stars in this sample, it does form an independent basis for testing the models. Figure 7*a* shows the comparison between the data of King (as published in Chiu 1980*a*) and the differential star counts predicted by the standard galaxy model; Figure 7*b* compares the color distributions. In both cases, the agreement is satisfactory. We have not used the color-distributions of King for fainter stars since a plot of the published colors given by Chiu (1980*a*) for these stars suggests that the fainter distributions are very noisy.

IV. SA 68: $b = -46°, l = 111°$

a) *Comparison with Data of Koo and Kron: $(J + F)/2 = 19.95$ to 22.15*

Figures 8*a* and 8*b* compare the color histograms in *J* and *F* of 623 faint stars observed by Koo and Kron (1982) with the distributions calculated using the standard galaxy model and spheroid color-magnitude diagrams like 47 Tuc (Fig. 8*a*) and M92 (Fig. 8*b*). The agreement is satisfactory in both cases. The breakdown of the predicted color distributions into separate model components is shown in Figure 8*c*, which was calculated using a spheroid color-magnitude diagram like M13. The spheroid subgiants and giants peak at about $J - F = 0.5$, where they contribute $\sim 25\%$ of the total number of stars. Evolved stars are relatively unimportant for this data sample, which, according to the models, mainly contains faint main-sequence stars. Figure 8*d* compares the original Kron (1978, 1980) data with the color-distribution obtained using a spheroid color-magnitude diagram like that of 47 Tuc.

b) *Comparison with Data of King: $V \leq 19.5$*

King has obtained star counts and colors in a small field of size 0.1 deg^2 (see Chiu 1980*a*). The counts for all 161 stars in this region and magnitude range are shown in Figure 9*a*. The agreement is satisfactory, although there are rather large fluctuations in the data because of small number statistics. Just as for the corresponding Figure 4*a* referring to SA 57, we see that disk giants and subgiants are mainly important in SA 68 for $m_V \leq 9$ and spheroid giants and subgiants are important for $m_V < 18$.

Figures 9*b* and 9*c* compare the observed and calculated color histograms in $B - V$; the agreement is excellent. The model distribution shown in these figures was derived using a spheroid color-magnitude diagram like that of 47 Tuc. The agreement is somewhat less satisfactory (see Fig. 9*d*) if a color-magnitude diagram like that of M13 is used (the spheroid peak model is bluer than in the observations by ~ 0.1 mag), although it is acceptable according to the criteria described in § II*c*.

The King and the Koo-Kron data for SA 68 are complementary: the King data cover the apparently brighter stars, and the Koo-Kron data contain the apparently fainter stars. Together they form a rather good coverage of the stars in SA 68 and constitute a useful test of the models.

V. SA 51: $b = 21°, l = 189°$

The field SA 51 is of special interest since it is almost exactly in the anticenter and is at a relatively low galactic latitude. Thus, the spheroid is expected, on the basis of

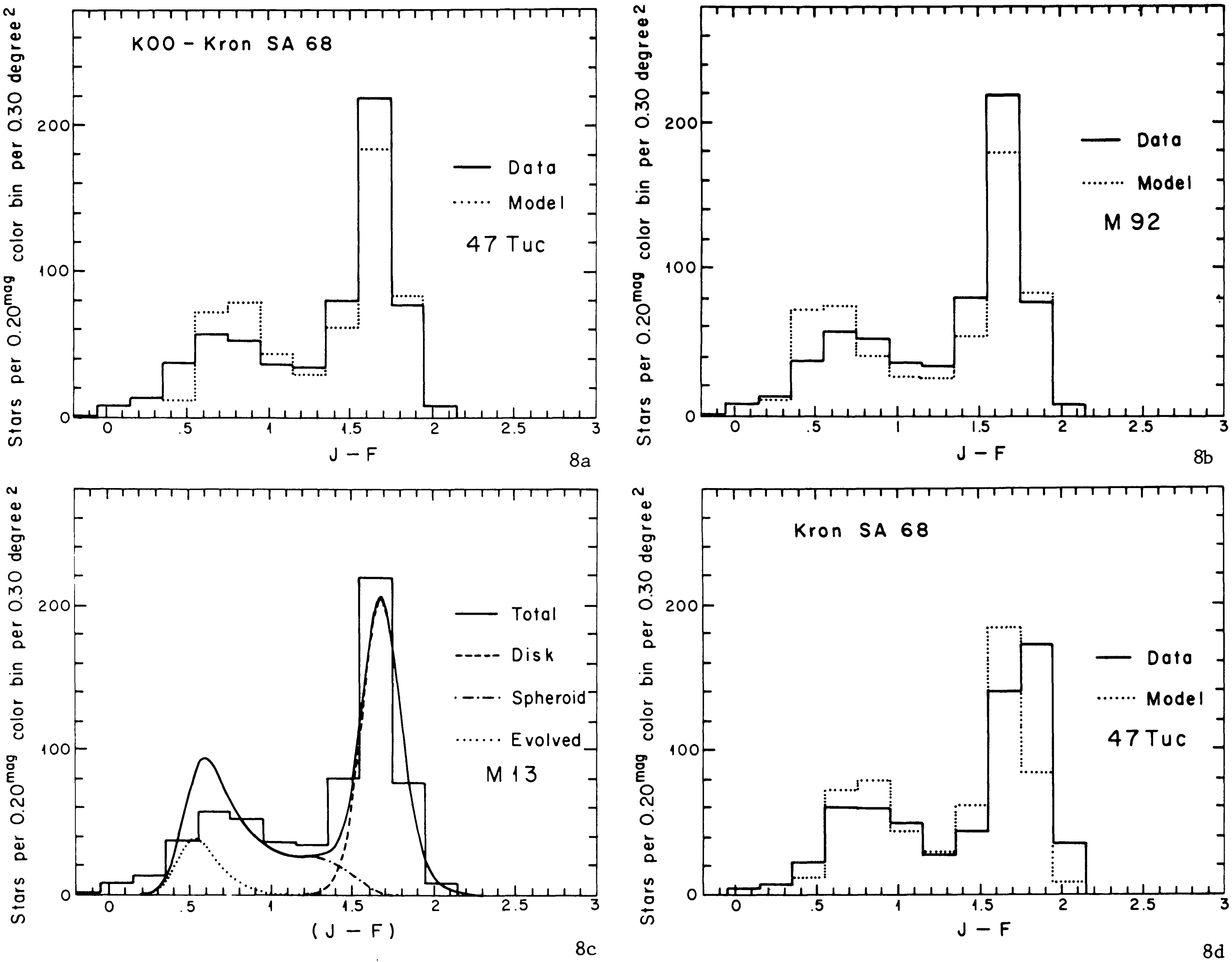


FIG. 8.—Comparison with color distributions of faint stars in SA 68 obtained by Koo and Kron. (a)–(b) Comparison of the color histogram of faint stars observed by Koo and Kron (1982) with the distribution calculated using the B&S model and a color-magnitude diagram like 47 Tuc for the spheroid (a) or a spheroid color-magnitude diagram like M92 (b). (c) Breakdown into individual components; the spheroid was assumed to have a color-magnitude diagram like M13 for this illustration. The contribution of subgiants and giants is labeled "evolved." (d) Comparison of the original Kron (1978, 1980) data with the color-distribution obtained using a spheroid color-magnitude diagram like 47 Tuc.

conventional galaxy models, to be unimportant for this field. SA 51 should provide, therefore, a good test of the model for disk stars. Chiu (1980b) inferred an unexpectedly large spheroid contribution to King's star counts for this field. The basis of Chiu's analysis has been questioned in Paper IV.

King's star counts (see Chiu 1980a) are compared in Figures 10a and 10b to the calculated star counts from our standard model assuming the Wielen (1974) and Luyten (1968) luminosity functions, respectively (see the discussion in § IIIa and also Fig. 1). The theoretical and observational results are in excellent agreement when the Wielen luminosity function is used. The agreement is less satisfactory, especially at fainter apparent magnitudes, when the Luyten function is used.

The color distribution observed by King for stars brighter than $m_V \leq 19.5$ is compared in Figures 10c and 10d with the distributions calculated using Wielen and Luyten luminosity functions. The total number of stars brighter than $m_V \leq 19.5$ is 468. The observed color distribution shows clearly that the Wielen function—with the dip near $M_V = 7$ (see Fig. 1a) or $B-V \approx 1$—is indicated by the data.

We conclude that both the star counts and the color distribution observed by King provide evidence for the dip in the luminosity function that was discussed by Wielen (1974) and, most recently, by Upgren and Armandroff (1981) (see Fig. 1a). This result is significant since it shows that the Wielen feature is present in the luminosity function of disk stars that are at a characteristic distance of 0.6 kpc from the solar position.

Figure 11a shows the separate contributions in the model from the disk and the spheroid to the color distributions of

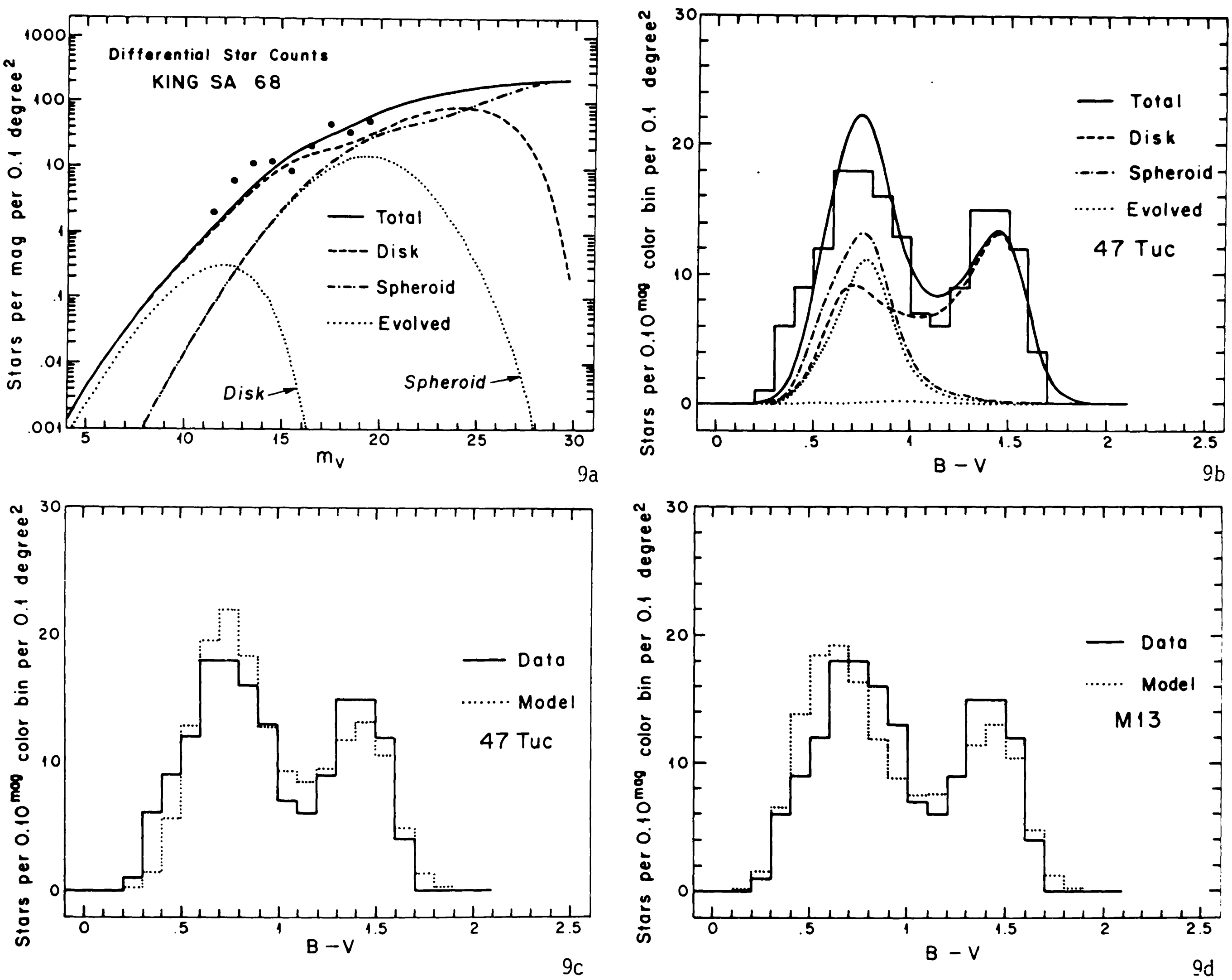


FIG. 9.—Comparison with King's star counts and color distributions in SA 68. (a) Comparison of observed and calculated star counts as a function of visual magnitude. (b)–(c) Comparison of the observed color histogram with the distribution of colors predicted by the B&S model; the spheroid color-magnitude diagram was assumed to be the same as for 47 Tuc in these calculations. (d) Comparison of the observed distribution and the calculated color distribution obtained with a spheroid color-magnitude diagram like M13. The magnitude range for stars included in (a)–(d) is $m_V \leq 19.5$.

stars brighter than $m_V = 19.5$ mag. The spheroid is seen to be very small in the model. There are, however, sharp features (0.1 mag wide) in the data at about $B - V = 0.5$ and 1.3 mag that are not present in the model. In both cases, the features are not present in adjacent bins and represent $\sim 2\ \sigma$ fluctuations in the stars in a given bin. We cannot explain such sharp features with our model and must regard them as statistical fluctuations. A further observational study of this field using a larger area than was investigated by King could confirm or reject this conjecture.

In all of the preceding discussion, we have followed Chiu (1980b) in assuming a total absorption of 0.2 mag. In order to investigate the importance of this assumption, we have calculated the color distribution for the 158 stars brighter than $m_V \leq 17$ both with and without absorption. The calculated colors are compared in Figures 11b and 11c with the observed distribution. The agreement is again excellent in both cases, with only minor differences introduced by the absorption. Figure 11d shows a similar comparison for stars brighter than $m_V = 19.5$ and no absorption (see Fig. 10c). Only minor differences are caused by the 0.2 mag of absorption in any of the examples we have considered.

The comparison of the calculated color distributions with and without absorption (Figs. 11b and 11c, 10c and 11d) reveals an interesting fact. The red stars are more affected by absorption than are the blue stars. The reason is that we are working with a magnitude-limited sample. Most of the blue stars are much brighter than the magnitude limit; hence, they are not much affected by 0.2 mag of absorption. The red stars are, however, intrinsically fainter, and many of them are near the magnitude limit. The absorption puts some of the faint red stars beyond the magnitude limit, while the bright blue stars remain well within the limit.

The main results that are implied by King's observations of SA 51 are (1) the Wielen (1974) luminosity function is a better fit to the data than the smooth Luyten (1968) function and (2)

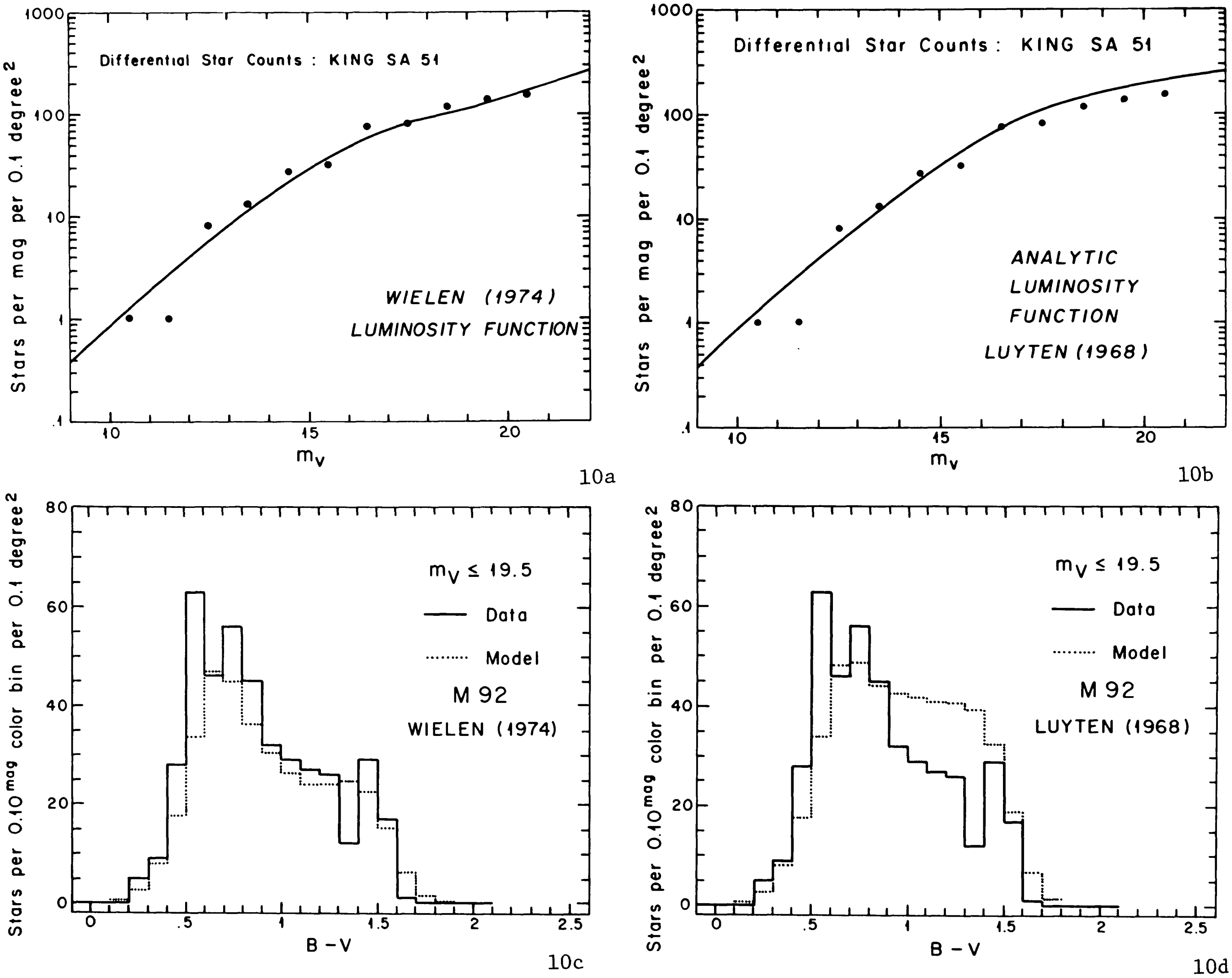


FIG. 10.—Wielen vs. Luyten luminosity functions for King's data in SA 51. (a)–(b) Comparison of the observed differential star counts with those calculated using a Wielen (1974) or a Luyten (1968) luminosity function. (c)–(d) A similar comparison for the color distribution. The observations indicate that the Wielen luminosity function is preferred.

the spheroid contribution is indeed small, as expected, in this nearly anticenter direction.

VI. AQUARIUS: $b = -51^\circ.1, l = 36^\circ.5$

Star counts and colors have been measured by Morton and Tritton (see Krug, Morton, and Tritton 1980; Morton and Tritton 1982; Tritton and Morton 1983) to $B = 20$ mag in an area of 0.31 deg^2 in a region of low interstellar extinction in the constellation of Aquarius. The sample is believed to be complete to at least $B = 19$ mag; to this limit, 399 stars are observed. The Aquarius field favors the detection of spheroid over disk stars because of its relatively small galactic longitude and relatively high galactic latitude.

Figure 12a compares the differential star counts measured by Tritton and Morton (1983) (see Paper VI; the refined star counts for this area will be published by Tritton and Morton 1983) to the calculated star densities obtained from the standard B&S model. The agreement is satisfactory, given the relatively small number of observed stars. Note that essentially all of the spheroid stars are predicted by the model to be subgiants or giants for $V \leq 18$ mag.

Figures 12b and 12c compare the color histogram measured for $B \leq 19$ mag by Morton and Tritton (1982) and Tritton and Morton (1983) with the color distribution predicted by the galaxy model. For Figures 12b and 12c, a spheroid color-magnitude diagram like that of 47 Tuc was assumed. The agreement is good. Figure 12b shows the contributions of the individual components in the galaxy model to the total calculated color distribution. The spheroid is more important than the disk for $B - V \leq 1.0$ mag, in this magnitude range. About 85% of all of the spheroid stars in the sample are predicted to be subgiants and giants.

Figure 12d shows the less satisfactory agreement that is obtained if the spheroid color-magnitude diagram is assumed to be like that of M13. The model predictions are shifted to the blue by ~ 0.15 mag, which does not agree as well with the

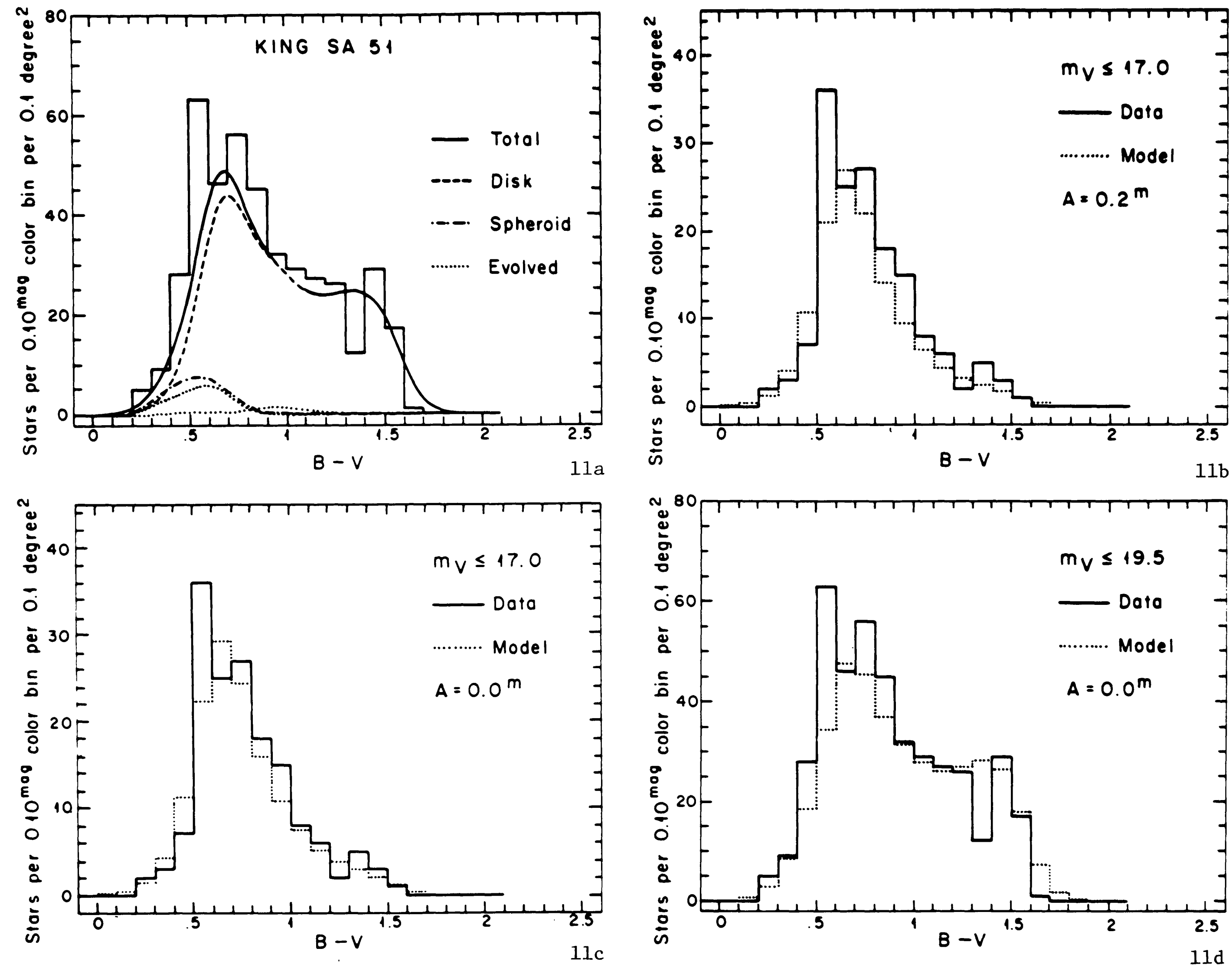


FIG. 11.—King's color distributions in SA 51. (*a*) Separate contributions of the disk and the spheroid to the color distribution. The spheroid contribution is small in the model. (*b*)–(*c*) Comparison of the observed distribution of 158 stars brighter than $m_V = 17$ with the distribution that was calculated assuming either 0.2 mag of absorption or no absorption. (*d*) The calculated distribution of colors for stars brighter than $m_V = 19.5$ with no assumed absorption. This figure should be compared with Fig. 10*c*. The effect of absorption is small.

data as the results that were obtained using the 47 Tuc color-magnitude diagram.

The present analysis of the Tritton-Morton data improves upon our earlier discussion (Paper VI) in taking account of the probable oblateness of the spheroid distribution (see eq. [1]) and in using the Wielen luminosity function (see Fig. 1) instead of the analytic luminosity function given in Paper I. We have also used the slightly improved data of Tritton and Morton (1983). The conclusions stated in the earlier discussion are verified here with the improved analysis.

The main new result that is obtained here is that the normalization of the spheroid which was determined in § II*d* using the data of Kron (1978, 1980) for SA 57 and by Koo and Kron (1982) for SA 68 also fits the Tritton-Morton data for the Aquarius field. This satisfactory agreement in the normalization, to an accuracy of order 25% or better, is seen most clearly in Figure 12*c*. The better agreement with the data that is obtained using the color-magnitude diagram for the spheroid of the metal-rich globular cluster 47 Tuc is consistent with the sense of the metallicity gradients observed in other galaxies (see, e.g., Pagel and Edmunds 1981): the metallicity is apparently higher in the direction toward the galactic center.

VII. SOUTH GALACTIC POLE

a) *Gilmore-Reid Data*

The Reid and Gilmore (1982) survey in the I_{RG} band of stars near the south galactic pole contains a large amount of data. However, one must make uncertain assumptions about

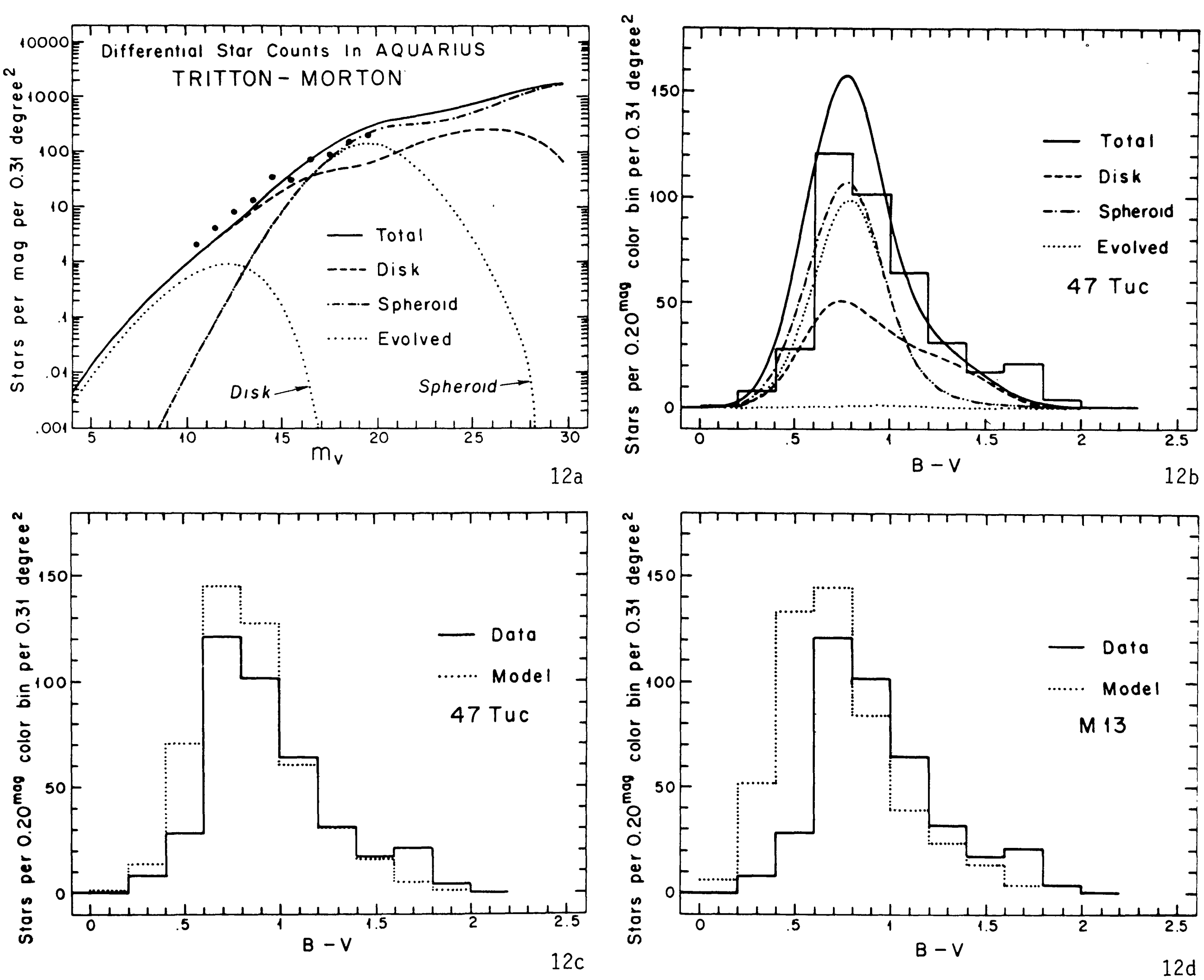


FIG. 12.—Comparison with the Tritton-Morton star counts and color distributions in Aquarius. Total number of observed stars is 399. (*a*) Comparison of observed and calculated differential star counts. (*b*)–(*d*) Comparison of the observed and calculated color distributions to $V \leq 19$. A spheroid color-magnitude diagram like that of 47 Tuc was assumed in the calculations that are described by (*b*) and (*c*); a color-magnitude diagram like that of M13 was used for (*d*). Following Tritton and Morton (1983), we have assumed a color dispersion in $B-V$ of 0.15 mag.

the transformation between their $V-I_{RG}$ colors and $B-V$ colors in which the B&S model is defined in order to analyze the data. Making the transformations recommended by Reid and Gilmore (1982) and Gilmore and Reid (1983), we obtain good agreement between their data and the star distributions calculated with the B&S model. Gilmore and Reid (1983) assumed that all of the observed stars were on the main sequence in analyzing their data using our model. For the magnitude and color range to which the Reid-Gilmore data apply, the B&S model actually implies (see Figs. 13*a*, 13*b* and 13*d*) that nearly all of the spheroid stars have evolved off the main sequence. This result is consistent with the color-magnitude diagrams of globular clusters shown in Figure 2 and the data of Ratnatunga (1982), which separate giants from dwarfs in the direction of the SGP (see § VII*b* below). Using color-magnitude diagrams of globular clusters to represent the spheroid color-magnitude diagram, we obtain good agreement with the observations of Reid and Gilmore (1982).

i) *Observations*

Reid and Gilmore (1982) carried out an extensive photographic survey of 18.24 deg² centered near the south galactic pole. Their primary data are three plates in the visual band (IIaD + GG495) and three in the I band (IVN + RG715). We denote the I band used by Reid and Gilmore (1982) by I_{RG}. The calibration of the Reid and Gilmore plates was established first in B and V by the intercomparison of photoelectric and photographic standards from a variety of sources. They made use of the apparently tight relation that exists between the (Cousins) $V-I$ color and the $B-V$ color for main-sequence disk stars in order to compute I magnitudes for all their standards for which only B and V were measured. The reader is referred to the paper of Reid and Gilmore (1982) for an informative discussion of many additional details that are omitted here of how the photometric calibration was carried out.

We have followed Reid and Gilmore (1982) and Gilmore and Reid (1983) in making the transformations from B and V magnitudes—in which our model is defined—into their color band. Unfortunately, much of the observational material necessary to make these transformations precise is not yet available. The disk main-sequence stars were transformed using the relation published by Reid and Gilmore (1982) between M_V and $V-I_{\text{Cousins}}$. There are no published transformation relations between $B-V$ and $V-I_{\text{Cousins}}$ for the color-magnitude diagrams of the globular clusters (which we use to represent the spheroid stars) and the evolved disk stars. There is a relatively tight relation between $V-I$ colors and $B-V$ colors that were measured photoelectrically by Johnson (1965, 1966). This relation is approximately independent of luminosity class in the color range of interest and is assumed (see Greenstein 1965; Gleise 1969; Eggen 1976) to be independent of metallicity (i.e., population) of the stars. The Johnson $V-I$ colors are converted to Cousins (1980) $V-I$ colors using the tabulated values (Johnson 1965, 1966; Keenan 1963) for the relation between M_V and $V-I_{\text{Johnson}}$ and the corresponding relation for Cousins colors that is given by Reid and Gilmore (1983). Finally, the Reid-Gilmore magnitudes, I_{RG}, are assumed to be in the Cousins I band for all the stars.

We have no way of estimating externally the systematic errors that exist in the Reid-Gilmore data or in the assumptions (described above) that must be made in order to translate our model from B and V to the observed plane of V and I_{RG}. *A priori*, we would expect that the systematic uncertainties are at least as large as the ones that were discussed in § II*c* for the more familiar color bands.

Table 4 of Reid and Gilmore (1982) gives the number–magnitude (in I_{RG})–color distribution ($V-I_{\text{RG}}$) for 11,324 objects that were classified by Reid and Gilmore as stellar in their field. The limit of this sample is $I_{\text{RG}} \leq 17.0$ mag. These are the data with which—with the aid of the above assumptions—we can compare our model calculations. A somewhat larger sample complete to $I = 18.0$ mag, but with $V \leq 19.0$ mag, of $\sim 12{,}500$ stellar objects was used by Gilmore and Reid (1983) in their discussion of the star counts, but the measured distributions of apparent magnitudes and colors were not given for this fainter sample. Gilmore and Reid (1982) published only the distributions after they made the conversion to absolute magnitudes assuming that all the stars were on the main sequence. Since we do not want to make this assumption, we cannot use their fainter sample.

ii) *Calculated versus Observed Distributions*

Figure 13*a* compares the numbers of stars versus apparent I_{RG} magnitude predicted by the standard B&S model with the numbers observed by Reid and Gilmore (1982). The parameters of the B&S model were fixed at the standard values described in § II. The agreement between the calculated and the observed counts is excellent.

Nearly all of the spheroid stars that contribute to the number counts by Reid and Gilmore are predicted to be evolved stars according to the calculations used in constructing Figure 13*a* (see also Figs. 13*b* and 13*d*). For the specific calculations on which Figure 13*a* is based, the color-magnitude diagram of the spheroid was chosen to be that of M13. Similar calculations were also carried out using the color-magnitude diagrams of M92 and 47 Tuc. The calculated total counts in the I_{RG} band are the same in all cases to an accuracy of a few percent. We conclude that good agreement exists between the differential star counts of Gilmore and Reid and the B&S model as long as one adopts some reasonable globular cluster color-magnitude diagram for the spheroid.

Figures 13*b*–13*f* compare the distribution of colors observed by Reid and Gilmore (1982) with the distribution calculated for the B&S model (using the assumptions described above). The agreement is excellent, especially considering the theoretical and observational uncertainties that are described above.

The color distributions that were calculated using the color-magnitude diagrams of M13 and M92 are closer to the observed distribution than the color histogram that is obtained assuming the spheroid resembles 47 Tuc. Figures 13*b* and 13*c* were generated using a spheroid color-magnitude diagram like that of M13; Figures 13*d* and 13*e* were constructed using the color-magnitude diagram of M92 for the spheroid. Figure 13*f* was constructed assuming that the spheroid color-magnitude diagram is like that of 47 Tuc. In all cases that are shown, we have convolved the model predictions for the number of stars versus color with a Gaussian function having a width of $\sigma = 0.15$ mag in $V-I_{\text{RG}}$ color. This smoothing function corresponds to a total dispersion which is made up of the intrinsic spread in colors of spheroid stars of a given absolute V magnitude, of measuring errors, and of scatter in the transformation relations between $B-V$ and $V-I_{\text{RG}}$.

All of the figures show the importance of evolved stars in the sample of Reid and Gilmore. Independent of whether the color-magnitude diagram of the spheroid is assumed to resemble that of M13, M92, or 47 Tuc, most of the spheroid stars in the model are evolved stars. The contribution of the evolved stars is particularly conspicuous near the peak in the observed distribution $V-I_{\text{RG}} = 0.6$ mag (see Figs. 13*b*, 13*d*).

Gilmore and Reid (1983) compared their observations with our model predictions for the number of stars per absolute visual magnitude interval. *Assuming that all of the stars were on the main sequence*, they concluded (see their Fig. 8) that their observations and our model are in "manifest disagreement." Their comparison was made in the apparent magnitude interval $15 \leq V \leq 17$.

We make a comparison in Figure 14 similar to that of Gilmore and Reid (1983), but we use a color-magnitude diagram like that of M13 for the spheroid stars. Figure 14*b* refers to the magnitude interval $0 \leq I_{\text{RG}} \leq 15$, and Figure 14*a* refers to the fainter interval $15 \leq I_{\text{RG}} \leq 17$. The agreement between our model and the published observations is *excellent* in both magnitude intervals.

We conclude that the published observations of Reid and Gilmore (1982)—both in total counts in the I_{RG} band and in $V-I_{\text{RG}}$ colors, are in good agreement with the B&S model, provided that one uses a color-magnitude diagram for the spheroid that is similar to what is observed for globular clusters. The disagreement with our model that was claimed by Gilmore and Reid (1983) was a result of their assumption that all the stars are on the main sequence.

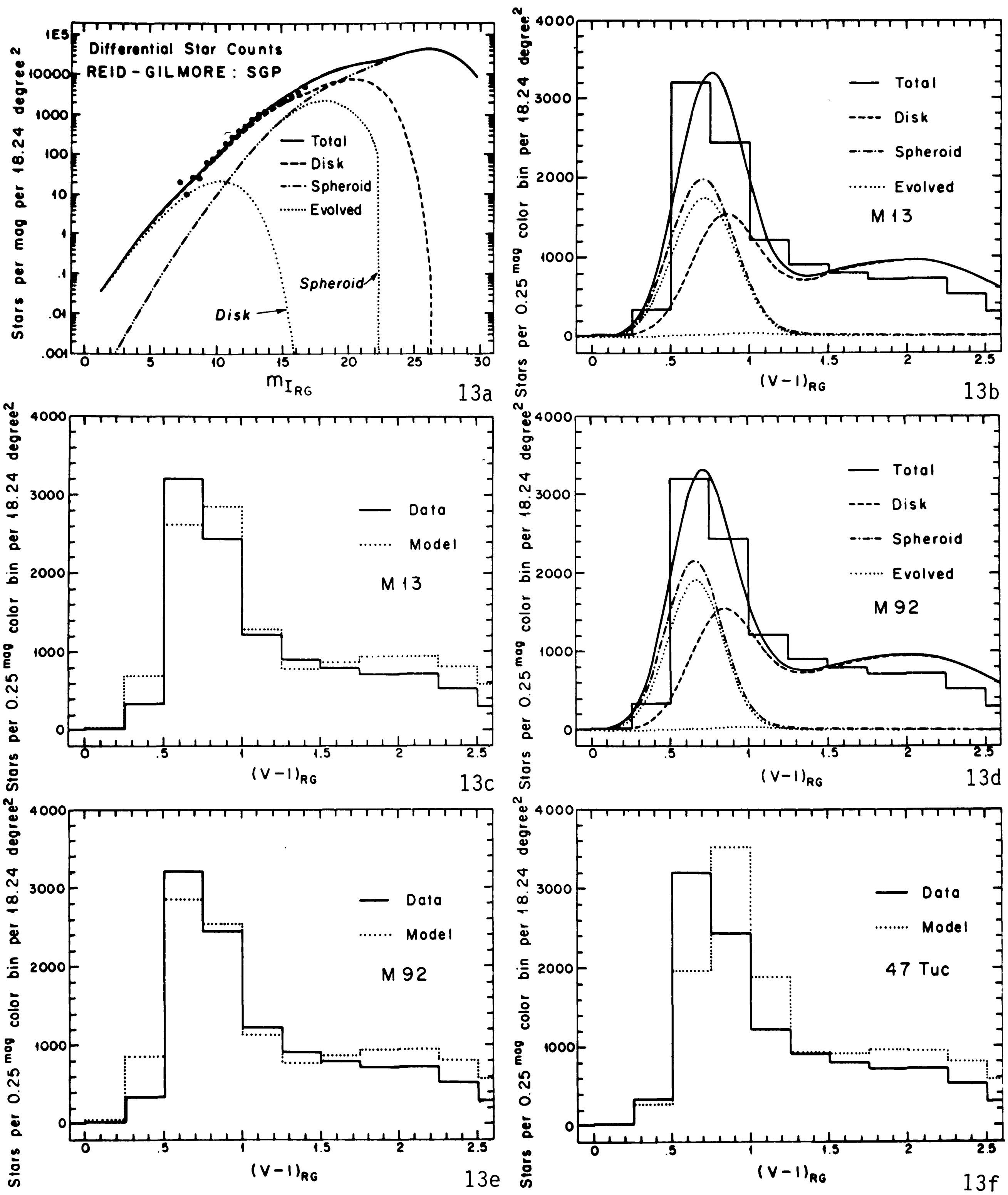


FIG. 13.—Comparison with the Reid-Gilmore survey at the south galactic pole. Observations in V and I_{RG} by Reid and Gilmore (1982) are compared with the distributions predicted by the standard B&S model. (a) Comparison of the differential star counts. (b)–(f) Comparison of the color distributions.

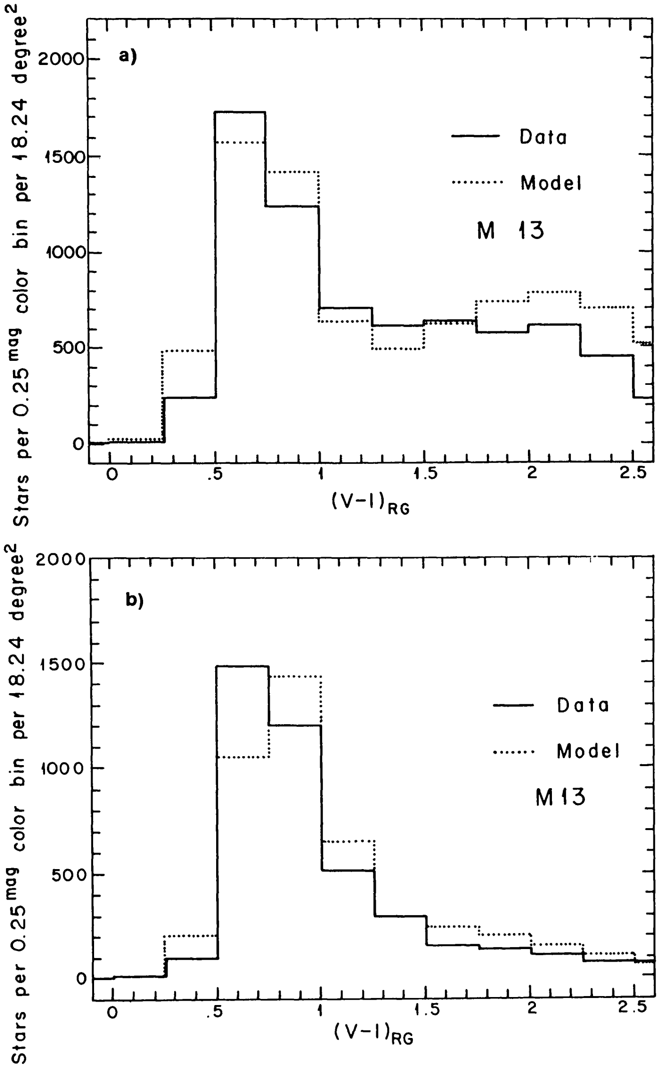


FIG. 14.—Comparison in separate apparent magnitude ranges with the Reid-Gilmore survey at the south galactic pole. Comparison is the same as in Fig. 13*c*, except that the results are broken down into two apparent magnitude ranges. (*a*) Apparent magnitude range $15 \le I_{RG} \le 17$ mag; (*b*) $I_{RG} \le 15$ mag. The agreement is excellent in both apparent magnitude ranges (cf. Fig. 8 of Gilmore and Reid 1983).

b) *Ratnatunga Preliminary Survey*

Ratnatunga (1982) has reported the results of low-resolution spectrophotometric observations of all stars with colors $0.9 < B - V < 1.4$ and visual magnitudes in the range $13 < V < 16$ in 2.8 deg^2 in SA 141, which is close to the SGP ($b^{II} = -86°$, $l^{II} = 246°$). Ratnatunga made use of the photometry of Bok and Basinski (1964). The Mg H and Mg *b* feature near 5100 Å was used to discriminate between spheroid giants and disk dwarfs (see also Clark and McClure 1979).

The results of Ratnatunga's survey are shown in Table 4, for both $13 < V < 15$ and for $13 < V < 16$, with $B < 16.9$ for $15 < V < 16$.

The expected number of disk and spheroid stars in both magnitude ranges have been calculated using the standard galaxy model with color-magnitude diagrams for the spheroid like those of M13, M92, and 47 Tuc. The calculated and observed numbers of stars are compared in Table 4.

TABLE 4

STARS IN 2.8 SQUARE DEGREES AT SOUTH GALACTIC POLE WITH $0.9 < B - V < 1.4$[a]

Spheroid Color-Magnitude Diagram	Disk	Spheroid Giants	Distance > 10 kpc
	$13 \le m_V \le 15$		
Observed	58	16	3
M13	46	13	3.5
47 Tuc	46	34	2.7
M92	46	5	1.0
	$13 \le m_V \le 16$[b]		
Observed	103	17	6
M13	97	19	9
47 Tuc	97	19	9
M92	97	15	5

[a] Observed values from Ratnatunga 1982.
[b] $m_B \le 16.9$ for $15 \le m_V \le 16$.

The observed and the calculated numbers of both disk and spheroid stars are in good agreement for the complete sample that refers to stars with $13 < V < 15$. Ratnatunga (1982) reached a similar conclusion based upon a less detailed version of the B&S model. The observed number of spheroid stars lies between the values predicted with an M13 and a 47 Tuc color-magnitude diagram, being closer to the value obtained for an M13-like diagram.

The number of spheroid stars that are expected to be found (in the magnitude-limited survey) at distances greater than 10 kpc from the Sun is predicted by the model and is also shown in Table 4. The number of such distant stars observed by Ratnatunga is also in agreement with the expectations based upon the model. These stars must have $M_V \le 0.0$ in order to be at the distance indicated by Ratnatunga (1982).

The proof by Ratnatunga of the existence of bright spheroid giants conflicts with (see § VIII below) the claim by Gilmore and Reid (1983) that the spheroid luminosity function cuts off sharply brighter than $M_V \approx 4$. *The large number of spheroid giants found by Ratnatunga (1982)—in agreement with calculations based on the B&S model—shows that the assumption by Gilmore and Reid (1982) that all the stars are on main sequence is incorrect.*

We conclude that the spheroid color-magnitude diagram in the direction of the SGP can be approximated by a combination of M13 and 47 Tuc diagrams, with the former having most of the weight.

VIII. SPHEROID LUMINOSITY FUNCTION

a) *Does the Luminosity Function Steepen Sharply Brightward of $M_V = +4$?*

Figure 1 shows the spheroid luminosity function that we have used in comparisons with the observed star counts and color distributions in the five fields discussed in this paper.

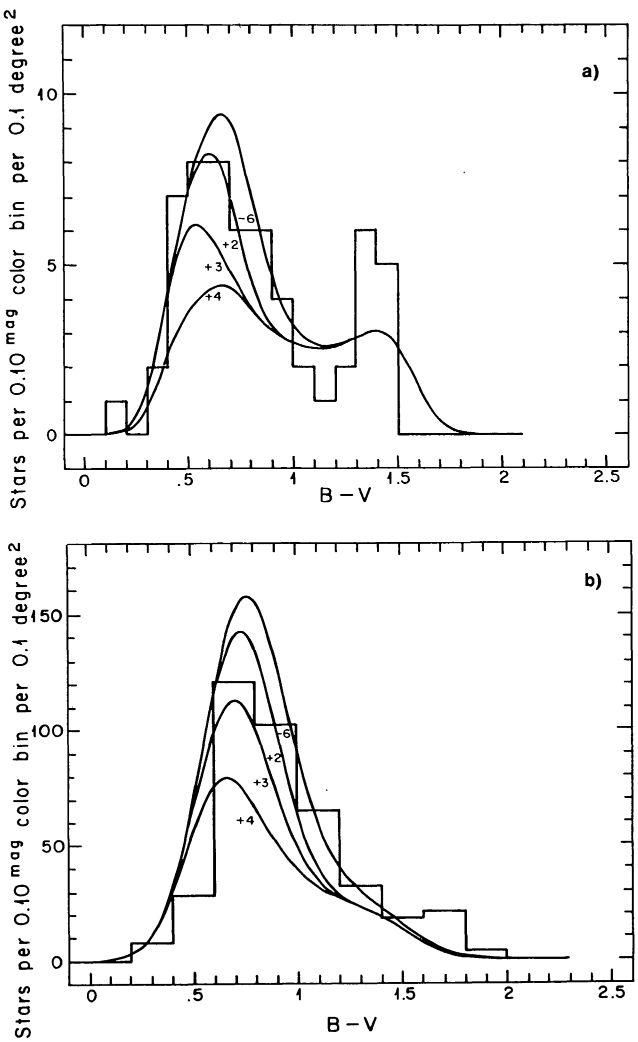


FIG. 15.—Bright end of the spheroid luminosity function. (*a*) Comparison of data of King (see Chiu 1980*a* and Fig. 7*b*) at the north galactic pole with a series of curves computed using different cutoffs for the spheroid luminosity function (and a M13–like color-magnitude diagram). Data contain stars to $V = 18$ mag. (*b*) A similar comparison with the data of Tritton and Morton (1983) for the Aquarius field. The spheroid color-magnitude diagram was assumed to resemble 47 Tuc (see Figs. 12*b* and 12*c*); the observations extend to $B = 19$ mag.

Gilmore and Reid (1983) have concluded that the spheroid luminosity function steepens sharply brightward of $M_V \approx 4$. However, they assumed—in order to infer absolute magnitudes—that all the stars were on the main sequence. In this subsection, we explore the question of whether there is evidence for a steepening of the spheroid luminosity function that does not assume a main-sequence color-magnitude diagram for all the stars and whether the suggestion of Gilmore and Reid (1983) is supported by the data of other observers.

Figure 15*a* shows the data of I. King at the north galactic pole that were previously discussed in § II*d* (see also Chiu 1980*a*). The continuous curves shown in Figure 15*a* are the predicted model color distributions when the spheroid luminosity function is assumed to be equal to zero for values brighter than $M_V = -6$, $+2$, $+3$, and $+4$. (For the extreme case in which the luminosity function is extended down to $M_V = -6.0$ mag, we have assumed that the disk and spheroid functions are similar in shape below $M_V = -2.0$ mag.). Figure 15*b* makes a similar comparison between the data of Tritton and Morton (1983) and the model color distributions.

It is clear from Figure 15 that the spheroid luminosity function must extend down to at least $M_V = +4$ and probably to $+3$. We find no evidence for a sharp steepening of the spheroid luminosity function brighter than $M_V = +4$ (beyond that shown in Fig. 1 for our assumed luminosity function) in the data of any of the observers for any of the five fields that we have studied in this paper.

The most conclusive evidence that the spheroid luminosity function does not sharpen steeply below $M_V \approx 4$ is the survey of Ratnatunga (1982) in which bright spheroid giants were discovered in the numbers predicted by the B&S model with a luminosity function that extended at least to $M_V = -2.5$ (see Fig. 1).

We have recalculated the expected number of spheroid giants that would have been found by Ratnatunga (1982) had the spheroid luminosity function been cut off brighter than specified values of M_V. We find that agreement with Ratnatunga's measured number of spheroid giants is only possible if the spheroid luminosity function extends to at least $M_V = +1$. For cutoffs such as were suggested by Gilmore and Reid, the calculated number of spheroid giants is smaller—for any of the globular cluster color-magnitude diagrams considered—than the observed number (see Table 4) by *an order of magnitude*. For example, the color-magnitude diagram of 47 Tuc predicts the largest number of spheroid giants in Ratnatunga's sample. If the spheroid luminosity function is cut off at $M_V = +4$, the expected number of spheroid giants in the faint sample would only be 0.4 compared with the observed value of 17 (see Table 4).

b) *Blue Tip of the Horizontal Branch*

Table 5 compares the observed versus the calculated number of stars on the blue tip of the horizontal branch, i.e., stars with $B - V < 0.2$ mag. The observations are shown for the cases which provide the strongest constraints: the field in Aquarius studied by Tritton and Morton (1983), the study of SA 57 by Weistrop (1972), and the investigation of the south galactic pole by Reid and Gilmore (1982). The model calculations are shown for four illustrative cases, corresponding to assumed color-magnitude diagrams for the spheroid stars like those of M3 (Sandage 1982*b*), M13 (Sandage 1970), M92 (Sandage 1982*b*), and 47 Tuc (Lee 1977). The luminosity functions are taken from Simoda and Kimura (1968), Hartwick (1970), and Da Costa (1982), assuming—as suggested by the available observations—that about one-half of the stars in the absolute magnitude range in which the horizontal-branch stars appear are on the blue tip of the horizontal branch for M3, M13, and M92. For 47 Tuc, we have assumed an upper limit of ~10% for the fraction of stars in the range $0.5 \le M_V \le 1.5$ that are on the blue tip of the horizontal branch.

There are, proportionally, an order of magnitude more stars (see observed and calculated columns of Table 5) on the blue tip of the giant branch in the globular cluster color-magnitude

TABLE 5

BLUE TIP OF HORIZONTAL BRANCH, $B - V \leq 0.2$ MAGNITUDES

Field	Observed	Calculated with Color-Magnitude Diagrams: M3	M13	M92	47 Tuc
Aquarius .... $B \leq 19$	0[a]	42	28	23	≤ 2
SA 57 .... $12 \leq m_V \leq 16$	50[b]	342	180	147	≤ 16
South Galactic Pole ... $I_{RG} \leq 17$	33[c]	695	421	345	≤ 33

[a]Tritton and Morton 1983.
[b]Weistrop 1972, 1980.
[c]Reid and Gilmore 1982.

diagrams of M3, M13, and M92 than are observed among the field spheroid stars in the Aquarius field or at the north or south galactic poles. This result is consistent with the earlier discussion (see Paper VI) of the absence of very blue stars in the Aquarius field. The observations are consistent with a horizontal branch similar to what is observed for 47 Tuc (Hesser and Hartwick 1977; Lee 1977).

IX. DISK SCALE HEIGHTS AND SCALE LENGTH

a) *Disk Scale Heights*

Three of the five fields studied in this paper show the bimodal color distribution at intermediate apparent magnitudes that was first noted in Paper I.[5] This bimodal distribution permits us to study the disk contribution separately from the spheroid stars in fields in which they both appear. We exploit this separation in the present section to determine the best-fit values and the uncertainties for the scale heights of disk stars with absolute visual magnitudes satisfying $5 \leq M_V \leq 13.5$.

Table 6 summarizes our results. The first column gives the absolute magnitude range for which the observational results in a particular field are sensitive to the assumed scale height of disk stars. The second column indicates the limits on the scale heights that result from fitting the data in the field listed in the third column. The uncertainties are intended to correspond to $\sim 1\ \sigma$ limits (i.e., 25% discrepancies in the number of disk stars in a given direction and apparent magnitude range; see § II*c*). The observer whose data are of primary use in this comparison is listed in the fourth column.

We show in Figures 16 and 17 the comparisons with the data that were obtained for each field with the scale height of faint disk stars set at the effective 1 σ upper and lower limits. The reader can judge for himself the appropriateness of our error estimates by comparing Figures 16 and 17 with the better agreement that was obtained when the scale height of the faint disk stars was held fixed at 325 pc. In order to facilitate this comparison, the last column of Table 6 shows which figures correspond to which field and also indicates which earlier figures compare the same observations with the results calculated using the standard disk scale height.

We conclude that the scale height of the faint disk stars satisfies

$$H_{\rm disk} = 350 \pm 50 \text{ pc}, \qquad 5 \leq M_V \leq 13.5. \tag{4}$$

The calculated number of disk stars in a given direction is proportional to the product of the volume density—which we have assumed to be constant in deriving equation (4)—and the scale height. Figures 16 and 17 and Table 6 show that the fluctuations in the average volume density of disk dwarfs in the four directions considered are not large. Assuming that the scale height is constant and equal to 325 pc, we conclude that the fluctuations in the volume density satisfy

$$\frac{n_{\rm disk}}{n_{\rm solar\ vicinity}} = 1 \pm 0.15, \qquad 5 \leq M_V \leq 13.5. \tag{5}$$

This limit on the fluctuations refers to typical distances of order 0.5 kpc.

McLaughlin's (1983) data on the star counts between $m_V = +4$ and $+9$ mag provide the best constraint on the scale height of the giant stars. By calculations similar to those described above, we find

$$H_{\rm disk} = 250 \pm 100 \text{ pc}, \qquad -1 \leq M_V \leq 3 \text{ mag}. \tag{6}$$

b) *Disk Scale Length*

The disk scale length, h, that is defined in Table 2 can be determined by comparing the observed and calculated number

[5]The bimodal distribution is predicted to appear only in certain directions and at intermediate and faint apparent magnitudes when both disk and spheroid stars are observable. At bright apparent magnitudes, only disk stars are important in all directions. Both disk and spheroid stars are significant for observations at the galactic poles and other high galactic latitudes, but the spheroid stars are unimportant in the direction of the anticenter (see Fig. 10). Bimodality is observed for SA 57 and SA 68 (see Figs. 5, 8, and 9), but is neither expected nor observed in the Aquarius field or in SA 51 (see Figs. 10–12). The conditions for the appearance of the bimodal distribution are discussed in § III*h* of Paper I and in Paper V.

TABLE 6
SCALE HEIGHTS OF DISK DWARFS

Absolute Magnitude Range	Limits on Scale Height (pc)	Field	Observer	Figures
$10 \leq M_V \leq 13.5$	350 ± 50	SA 57	Kron	16*a*–16*b* cf. 5*b*
$9.5 \leq M_V \leq 13.0$	350 ± 50	SA 68	Koo-Kron	16*c*–16*d* cf. 8*b*
$4.5 \leq M_V \leq 10.0$	325 ± 50	SA 51	King	17*a*–17*b* cf. 10*c*
$5 \leq M_V \leq 11.5$...........	325 ± 75	SA 141	Reid-Gilmore	17*c*–17*d* cf. 13*c*

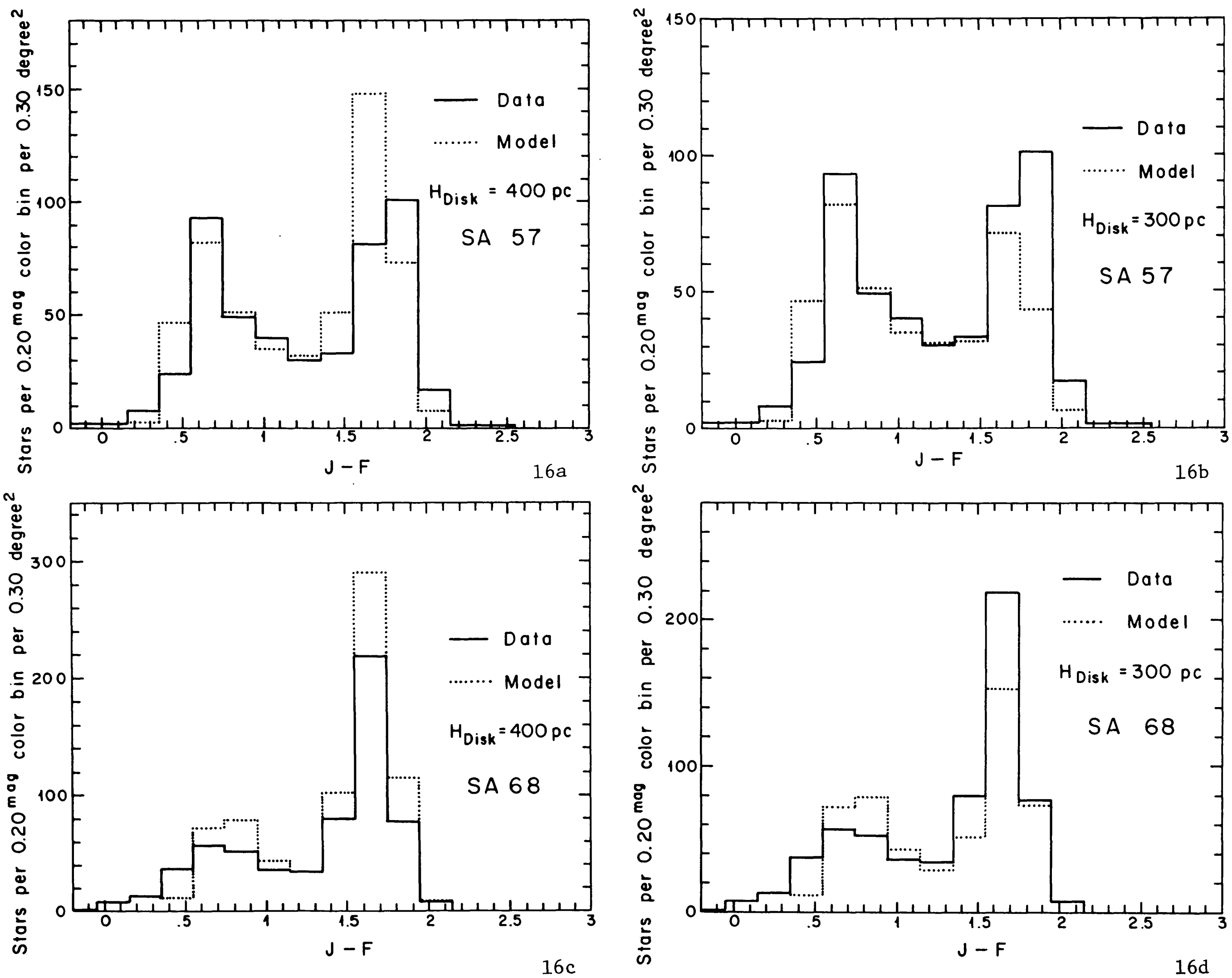


FIG. 16.—Disk scale heights in SA 57 and SA 68. (*a*)–(*b*) Comparison of color distributions calculated with extreme values for the disk scale lengths with the observations of Kron (1978, 1980) for SA 57. (*c*)–(*d*) A similar comparison with data of Koo and Kron (1982). Except for the disk scale height, the model parameters were the same in (*a*) and (*b*) as in Fig. 5*b*, and the same in (*c*) and (*d*) as in Fig. 8*a*.

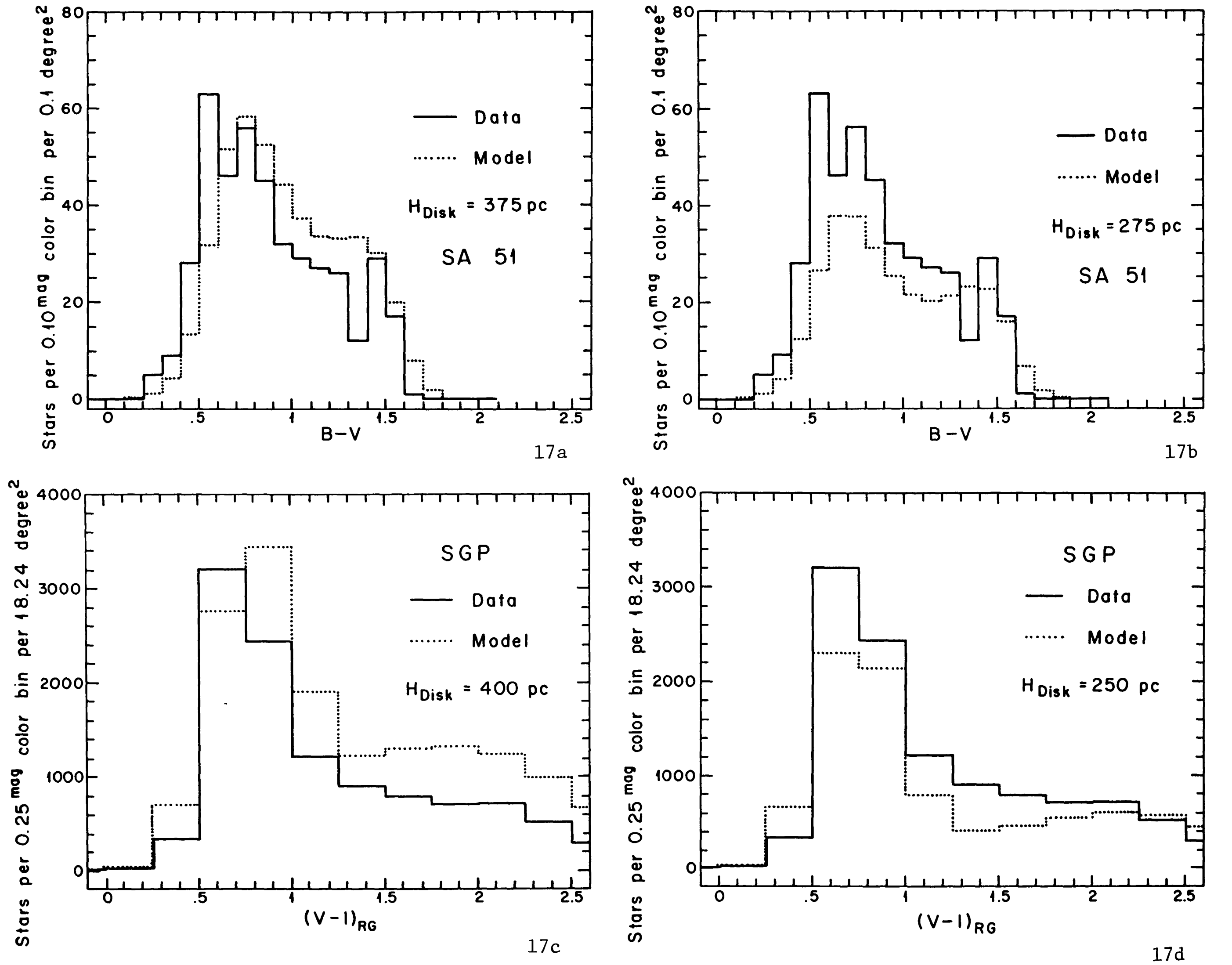


FIG. 17.—Disk scale heights in SA 51 and the south galactic pole. (*a*)–(*b*) Comparison of color distributions calculated with extreme values for the disk scale heights with the observations of King (see Chiu 1980*a*) for SA 51. (*c*)–(*d*) A similar comparison with data of Reid and Gilmore (1982). Except for the disk scale height, the model parameters were the same in (*a*) and (*b*) as in Fig. 11*d*, and the same in (*c*) and (*d*) as in Fig. 12*c*.

of disk stars at different galactocentric distances. The integrand of the expression for the differential star counts contains a factor

$$\text{integrand} \propto \exp\left(-R \cos b \cos l/h\right), \qquad (7)$$

where R is the distance from the Sun and b and l are the galactic latitude and longitude of the field of interest. The characteristic value of R is about $H_{\text{disk}}/|\sin b|$. One can always set a lower limit on h with observations of the disk component because the exponent in equation (7) can be made arbitrarily large (in absolute value) as h is decreased. However, for the available data, the exponent in equation (7) is not very large with h equal to the standard value (3.5 kpc); thus, setting h equal to a larger number does not make an appreciable difference. This is another way of saying that the available counts of disk stars do not reach sufficiently far from the Sun to set useful constraints on how large h can be.

We have carried out calculations of the expected number of disk stars in SA 51 (which is in the direction of the anticenter) and in Aquarius (which is in the first quadrant). Using again (see § II*c*) the criterion that the predicted and observed number of stars should not differ by more than 25%, we find that King's data for SA 51 (see Figs. 10 and 11 and § V) implies that 2.5 kpc $\leq h$. The Tritton-Morton (1983) data (see Fig. 12 and § VI) are somewhat less sensitive because their field is at higher galactic latitude (see eq. [7]). For the Aquarius field, we find 2.0 kpc $\leq h$. We are unable to set a useful upper limit on h.

We conclude that the disk scale length satisfies

$$2.5 \text{ kpc} \leq h. \qquad (8)$$

This result is consistent with the measurements of disk scale lengths of other galaxies of similar Hubble type to the Galaxy.

X. GILMORE-REID THICK-DISK MODEL

Gilmore and Reid (1983) proposed a thick disk model for the Population II stars in the Galaxy. The evidence for this model is presented in Figures 6*a* and 6*b* of their paper, which show that the number of observed stars versus *inferred* distance can be fitted by two exponential density laws.[6]

The evidence presented by Gilmore and Reid for their thick-disk model depends upon the assumption that *all* the observed stars are on the main sequence. The distances that were used in making the plots in Figure 6 of their paper were obtained by applying a main-sequence color-magnitude relation to the measured color of each star in their sample. The inferred absolute magnitudes were used together with the measured apparent magnitudes to calculate the distances.

We test the thick-disk model in three ways. We repeat the calculations of Gilmore and Reid (1983) and confirm that their model does indeed provide a good fit to their data on intermediate magnitude stars at the south galactic pole. We then apply their model to the observations of fainter stars in SA 57 by Kron (1978, 1980) and in SA 68 by Koo and Kron (1982). The thick-disk model disagrees with the data on fainter stars. Finally, we show that our standard model gives results similar to those inferred by Gilmore and Reid if all the model stars have their computed apparent magnitudes and colors, but the distances are incorrectly interpreted as a result of using only a main-sequence color-magnitude diagram.

We have followed Gilmore and Reid in defining a composite thick-disk model with two components: (1) our standard disk with a characteristic scale height for the oldest stars of ~ 350 pc; and (2) a thicker component that has a characteristic scale height of ~ 1450 pc. The ratio of the two components is indicated by the intercepts of the two straight lines in Figure 6*a* of Gilmore and Reid (1983) and corresponds to the thicker component having $\sim 2.5\%$ of the normalization of the traditional component at the solar position. Following a proposal by Gilmore (1983), we have used a Wielen (1974) luminosity function for the thick disk that is cut off near $M_V = +4$ with a Gaussian width of 0.75 mag in absolute visual magnitude. This cutoff is necessary so that the thick-disk model will not predict too many blue stars in the relatively bright apparent magnitude range studied by Reid and Gilmore (1982).

Figure 18*a* shows the excellent agreement that is obtained between the observed star counts of Gilmore and Reid (1983) and the results of a calculation based on their thick-disk model. Figure 18*b* shows that the thick-disk model also represents well the color distribution observed by Gilmore and Reid.

However, the predictions of the thick-disk model disagree with the observations of fainter stars in SA 57 and SA 68. This is shown in Figure 19*a* for SA 57 (Kron's 1978, 1980 data) and in Figure 19*b* for SA 68 (data of Koo and Kron 1982). The thick-disk model fails to reproduce the blue peak near $J-F=0.7$ mag. The disagreement would have been exacerbated had we included (see Gilmore and Reid 1983) a metallicity gradient for the thick-disk component since it would have moved the thick-disk contribution blueward by ~ 0.2 mag in $J-F$. This would place the thick-disk stars closer to the center of the valley between the two peaks in the observed star distribution (see the discussion in Paper IV, § VI, on intermediate scale height populations).

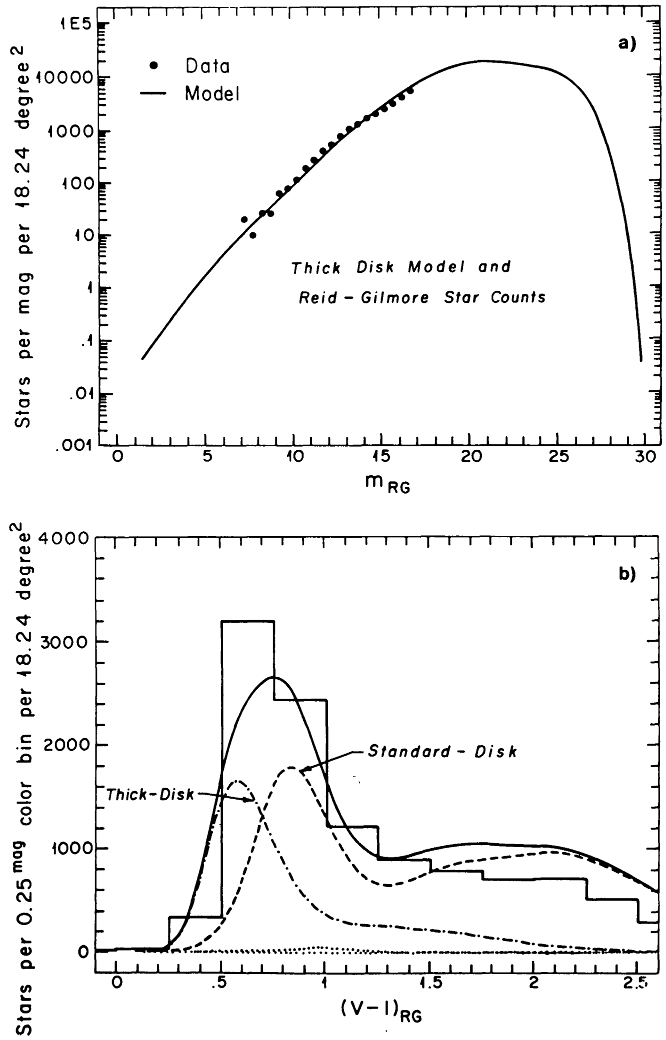


FIG. 18.—Thick-disk model compared with the Reid-Gilmore observations at the south galactic pole. (*a*) Comparison of star counts predicted with the aid of the Gilmore and Reid (1983) thick-disk model with the counts observed by Reid and Gilmore (1982). (*b*) Comparison of predicted color-distribution with the observed distribution. The histogram represents the data. The smooth curve represents the prediction of the total model. The agreement is satisfactory in both cases.

Figures 20*a* and 20*b* are simulated versions—with our standard galaxy model—of how Gilmore and Reid reduced their data. We computed a standard model in the visual band, V, and calculated the I_{RG} magnitudes following the precepts outlined in § VII. We then reinterpreted, following Gilmore and Reid, all of the individual stellar distances using their assumed main-sequence color-magnitude diagram. The results are plotted in Figure 20 in the same coordinates as were used by Gilmore and Reid and for the same pseudo-absolute visual magnitude ranges, $M_V = 4$–5 and $M_V = 5$–6. The straight

[6]An additional argument given by Gilmore (1983), which is based on the distribution of K giants perpendicular to the galactic disk, is discussed in Appendix B.

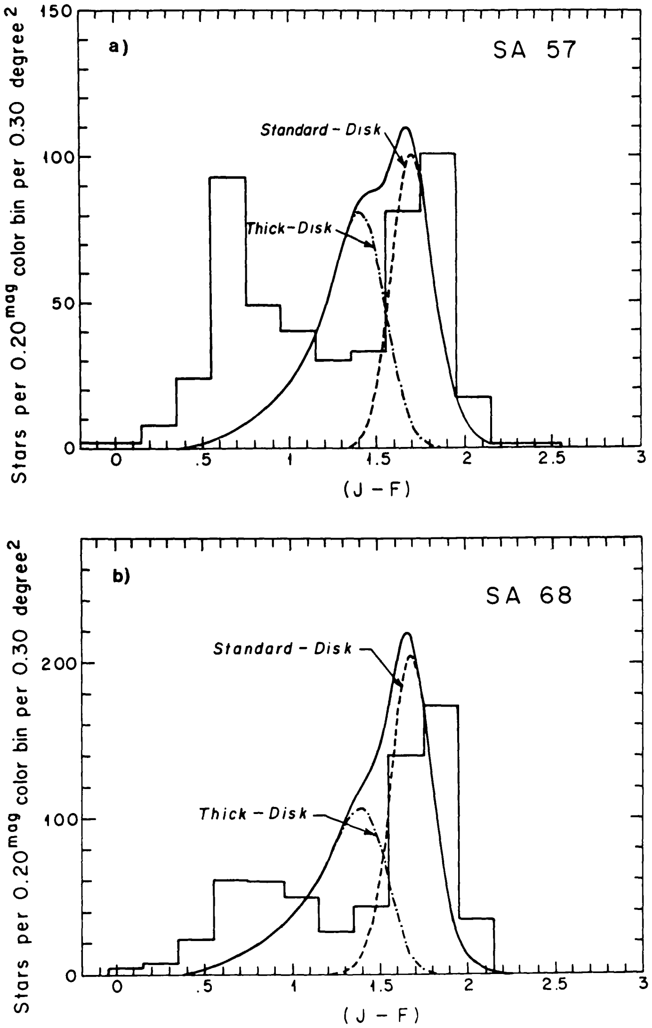


FIG. 19

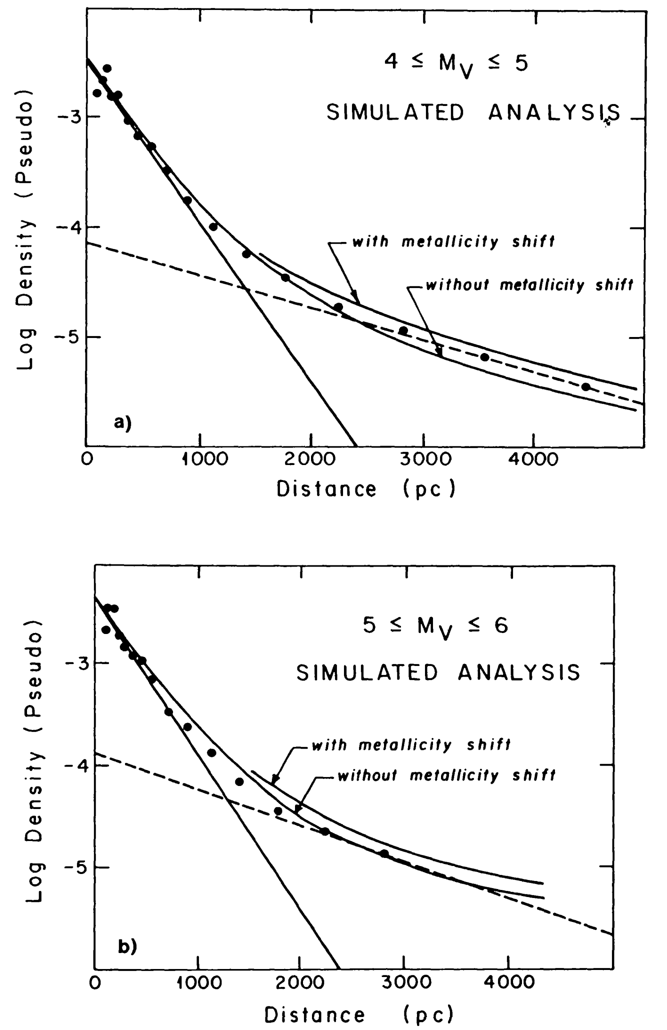


FIG. 20

FIG. 19.—Thick-disk model compared with the observations of fainter stars in SA 57 and SA 68. The histogram is the data. The smooth continuous curve represents the prediction of the total model. (*a*) Comparison of thick-disk model of Gilmore and Reid (1983) with the observations of Kron (1978, 1980) for SA 57. (*b*) A similar comparison with data of Koo and Kron (1982) for SA 68. The poor agreement obtained with the thick-disk model should be contrasted with the good agreement obtained with the standard disk and spheroid model as shown in Figs. 5 and 8 (see also §§ III*b*, IV*a*).

FIG. 20.—Simulation of the Gilmore-Reid analysis of B&S model. Density of stars inferred from the standard B&S model *assuming* that all the stars are on the main sequence, compared with Fig. 6 of Gilmore and Reid (1983) who originally made this assumption. Data points are from Gilmore and Reid. Curves refer to a model computed using an M13 color-magnitude diagram for the spheroid. Upper line contains a full metallicity shift of 1 mag downward for all the subdwarfs in the color-magnitude diagram (see Gilmore and Reid 1982); lower line was computed assuming no metallicity shift for the subdwarfs. (*a*) Except for the information representing our model, same as Fig. 6*a* of Gilmore and Reid and refers to (pseudo) absolute visual magnitudes between 4 and 5. (*b*) Except for data representing our model, same as Fig. 6*b* of Gilmore and Reid and refers to the absolute magnitude range between 5 and 6. Straight lines were drawn by Gilmore and Reid to represent the standard disk and the thick-disk components. Simulated analysis accurately reproduces the density distributions inferred by Gilmore and Reid.

lines in Figures 20 are taken directly from Figure 6 of Gilmore and Reid (1983); the data points are also taken from their paper.

The two curved lines in Figures 20*a* and 20*b* represent the results of our model when we reinterpret all the distances in terms of a main-sequence color-magnitude diagram. The upper line contains a full metallicity shift of 1 mag downward for all the subdwarfs in the color-magnitude diagram (see Gilmore and Reid 1983); the lower line was computed assuming no metallicity shift for the subdwarfs. The pseudo-densities of Figure 20 are well represented by a moderate metallicity shift.

Our standard model—suitably misinterpreted—fits well the density distributions that were inferred by Gilmore and Reid (1983). The thick disk is introduced artificially by the assumption that the giant stars are on the main sequence. Spheroid giants that are of order 10 kpc away in our model are assigned to a thick disk of dwarf stars (scale height ~1.5 kpc) by the Gilmore-Reid analysis.

XI. SUMMARY AND DISCUSSION

The main result of this paper is that a simple model—the standard galaxy model—fits all of the modern data on star counts and color distributions in five widely separated fields in the Galaxy. The observations extend over a large range in apparent magnitudes, five orders of magnitude in star densities, and several different color bands.

The two components of this model are a thin exponential disk and a de Vaucouleurs spheroid, both of which are defined quantitatively in Table 2. The luminosity functions and color-magnitude diagrams that are used in conjunction with these density laws are shown in Figures 1 and 2. Together they define the model in the B and V bands. Calculations for other bands require color transformations. The detailed comparisons with observations are made in §§ III–X, Figures 5–22 (see Appendix A for Figs. 21–22), and Tables 3–6.

Since the agreement with observations is good, we may use the model to set limits on some of the basic parameters of the Galaxy by comparing with the data. The principal results are summarized in Table 1.

We discuss below our major conclusions.

1. The accuracy of the data is an important consideration. Figure 3 and the discussion in § II*c* show that there are appreciable uncertainties even when one works in the familiar B and V bands and at relatively bright apparent magnitudes. The uncertainties in the observations may be even larger in magnitude bands for which there is less photometric experience and data.

We estimate uncertainties in the inferred model parameters by allowing the discrepancies to reach 0.2 mag in color shifts and 25% in normalization; the quoted uncertainties are intended to correspond approximately to 1 σ errors. We do not make a simultaneous solution by allowing all the parameters to vary. With the available data, this would not be justified. We do show in each case the comparison between observation and calculation upon which our conclusions are based so that the reader can judge for himself whether or not our estimates of uncertainties are appropriate.

Observations of the same field in the same magnitude range by groups using different reduction and analysis techniques would be very informative.

2. The scale height of disk dwarfs is constrained by observations of Kron in SA 57, of Koo and Kron in SA 68, of King in SA 51, and of Reid and Gilmore at the south galactic pole. Assuming that the Wielen (1974) disk luminosity function applies everywhere that we have data, we find that a consistent solution for all these fields *constrains the dwarf scale height to be in the range 350 ± 50 pc for* $5 \leq M_V \leq 13$. The results of this analysis are summarized in § IX*a*, Table 6, and Figures 16–17.

Using the same data, we infer that the number density of stars in the observable region ($5 < M_V < 13$) is equal to the value in the solar vicinity to an accuracy of ±15%, provided the scale heights are fixed at 325 pc.

The scale height of disk giants is less well determined. Using the data of McLaughlin (1983), we find that the giant scale height lies in the range 250 ± 100 pc.

The disk scale length exceeds 2.5 kpc, according to an argument similar to that given to constrain the scale heights (see § IX*b*).

3. Wielen (1974) and Upgren and Armandroff (1981) have shown that there is a dip in the luminosity function of *nearby* disk stars at about $M_V = 7$ (see Fig. 1*a*) or $B - V \approx 1$. Both the star counts and the color distribution observed by King (see Chiu 1980*a*) for the more distant stars in SA 51 provide evidence for this dip. The results are shown in Figure 10 and are discussed in § V. We conclude that *the Wielen (1974) luminosity function is a better fit to the data than the smooth Luyten (1968) function.*

4. The spheroid axis ratio at ~10 kpc from the galactic center can be determined by comparing star counts in the plane that is perpendicular to the direction to the galactic center, using a method that was first discussed in Paper I. An ideal experiment would involve three or more fields in this plane at high galactic latitudes and symmetrically placed about the galactic poles. With three fields, one could overdetermine the two important parameters: the spheroid normalization and the axis ratio. Systematic uncertainties in the data would be apparent from a comparison of the fields symmetrically placed about the galactic pole. High latitudes ($b \geq 45°$) would guarantee that obscuration is unimportant.

We have made a determination of the axis ratio using two fields: SA 57 (at the north galactic pole) and SA 68 (which is close to the desired plane). Our result is described in § II*d* and is $b/a = 0.80^{+0.20}_{-0.05}$. This result is in good agreement with the values of the axial ratio found at small galactocentric distances by Oort and Plaut (1975) and by Fall (1981) and also with the value obtained for the overall globular cluster system by Frenk and White (1981). We conclude that there is no evidence for a large gradient in the axial ratio of spheroid stars (see the discussion in § II*d*).

The normalization of the eccentric spheroid is determined by the same argument and is given in equations (2) and (3) of § II*d*. This normalization is then used throughout the paper to obtain satisfactory agreement with all of the observations.

The spheroid normalization given here is about 1/500 the number of disk stars estimated locally by Wielen (1974). The numerical values are given in Table 2 and in § II. The relative spheroid-disk normalization inferred here is slightly more than the factor of 1/800 that was deduced in Paper I assuming an axial ratio of 1.0.

5. There is a suggestion of a metallicity gradient in the data. The distances at which the maximum contribution to the spheroid star counts occur can be calculated with the aid of equation (8) of Paper IV. The characteristic distances for the spheroid stars are SA 57 (10.5 kpc), SA 68 (11.5 kpc), SA 51 (13.5 kpc), Aquarius (7.7 kpc), and the SGP (10.5 kpc). The field with the smallest galactocentric distance, Aquarius, is best described by a 47 Tuc-like color-magnitude diagram (see § VI). For SA 57, which probes intermediate galactocentric distances, the Kron and Weistrop data are best fitted by an M13 color-magnitude diagram (see § IV). These results are consistent with the observed decrease of metallicity with galactocentric distance in other galaxies. Unfortunately, the data are not yet adequate in any of the other fields to permit a decisive test of this expectation.

6. *Nearly all of the spheroid stars at intermediate magnitudes (brighter than about $V = 18$) are either giants or subgiants in our model.* This is a direct result of using globular cluster color-magnitude diagrams to model the spheroid. Examples are shown in many of the figures.

The calculation of the model predictions for the distribution of stellar colors is made computationally difficult by the sinuous nature of globular cluster color-magnitude diagrams. These diagrams must be represented by accurate (spline) fits that are then used to calculate the color distributions. There is no short cut that can be safely followed. Some previous analyses have incorrectly assumed that the model was in conflict with a particular piece of data because the computations using globular cluster color-magnitude diagrams were not made.

The spectroscopic survey by Ratnatunga (1982) confirms the existence of a large number of evolved spheroid stars, in agreement with the model. These results are described in § VII*b* and Table 4. No evidence is found for the sharp steepening of the luminosity function brighter than $M_V \approx 4$ that was inferred by Gilmore and Reid (1983). The data of King at the north galactic pole and of Tritton and Morton (1983) in the Aquarius field provide additional evidence supporting the existence of bright spheroid stars in the quantities predicted by the model. The evidence is described in § VIII*a* and Figure 15.

7. The observations in three fields (SA 57, the Aquarius field, and the south galactic pole) show that *the blue tip of the horizontal branch of the field spheroid stars is missing*. If the spheroid field stars had horizontal branches like M3, M13, or M92, then an order of magnitude more blue stars ($B - V < 0.2$ mag) would have been detected than were observed. These results are summarized in Table 5. The observations are consistent with the spheroid field stars having a horizontal branch similar to what is observed for 47 Tuc (Hesser and Hartwick 1977; Lee 1977).

8. *The thick-disk model of Gilmore and Reid (1983) disagrees — see Figure 19 — with the data of Kron (1978, 1980) for the faint stars in SA 57 and with the data of Koo and Kron (1982) for the faint stars in SA 68.* The predicted stars from the thick disk fall near the minimum of the color distribution between the spheroid and the disk peaks (see also Fig. 8 and § VI of Paper IV).

The B&S model fits well both the star counts and the color distribution measured by Reid and Gilmore (1982) in V and I_{RG}. *The previous claim by Gilmore and Reid that the standard galaxy model does not fit their data was a result of their assumption that all the stars were on the main sequence.* The excellent agreement between our model and the Reid-Gilmore data is shown in Figures 13 and 14.

We also applied the Gilmore-Reid analysis to a simulated Galaxy created with the aid of the standard galaxy model. We distributed stars—according to the model—in a thin disk and a de Vaucouleurs spheroid. We then applied the Gilmore-Reid assumption that all the stars are on the main sequence to derive pseudo-distances for the model stars from their colors and apparent magnitudes. The results are shown in Figure 20 and are discussed in § X. The main-sequence assumption implies the existence of a thick disk for our model stars even though this is not the distribution that we created in the model. The spheroid giants at a typical distance of 10 kpc are brought in to a distance of 1 or 2 kpc by the main-sequence assumption.

We conclude that the previously claimed disagreement between the B&S model and the data of Reid and Gilmore resulted from their assumption that all the observed stars were on the main sequence.

9. The available data are not sufficient to determine whether or not the feature in globular cluster luminosity functions that is labeled "GCF" in Figure 1*a* is also present in the luminosity function of the spheroid field stars. This conclusion is based upon comparisons described in Appendix A between predicted star distributions, computed with and without the GCF, and all of the available data. Future accurate observations at the galactic poles between $V \approx 16$ and 18 mag could establish to what extent the GCF exists in the luminosity function of the spheroid field stars (see Fig. 23).

10. The distribution of K giants perpendicular to the galactic plane is discussed in Appendix B. The densities predicted by the standard galaxy model are in excellent agreement with Oort's (1960) summary of the data out to 2 kpc. Gilmore (1983)—see his Figure 4—fitted the K giant densities by two exponential laws, a thin disk and a thick disk. *A similar fit to the densities predicted by the standard galaxy model — see Figure 24 — could be interpreted also as indicating a thick disk (with the properties inferred by Gilmore 1983), although no thick disk is present in the model.*

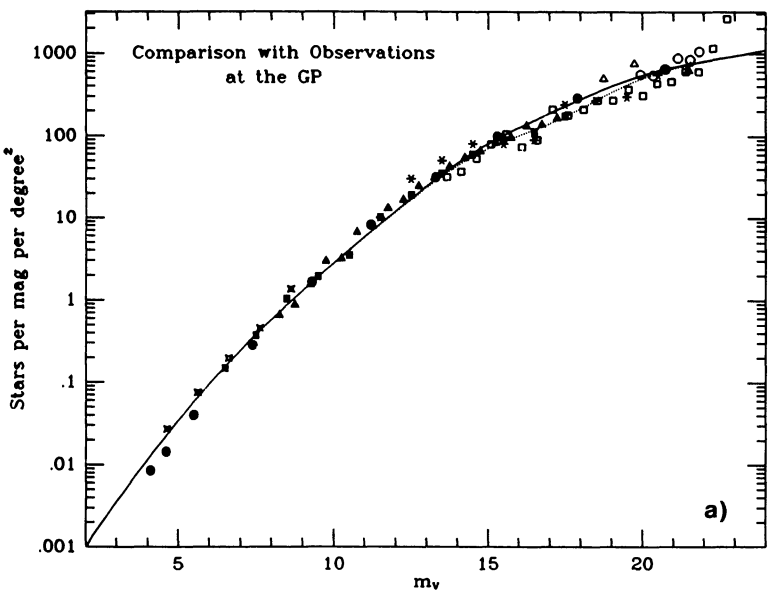
11. The quasar number density at faint magnitudes is constrained by the good agreement between the galaxy model and the observed color histograms for objects of stellar appearance (for early discussions of this argument see Bahcall and Soneira 1980*b*, 1981*a*; also Kron and Chiu 1981; Schmidt and Green 1983). Of all the color histograms shown in this paper, only the histograms of very faint stars ($[J + F]/2 \leq 22.15$ mag) by Kron (1980) for SA 57 (see Fig. 5) and Kron (1980) and Koo and Kron (1982) for SA 68 (see Fig. 8) show evidence for an excess population of blue objects. We conclude from these observations that the number of white dwarfs and quasars with $J - F \leq 0.35$ mag is $\lesssim 75$ deg^2 for $J + F \leq 22.15$ mag. Making a small correction (see Paper I; Bahcall and Soneira 1980*b*) for the blue white dwarfs ($\lesssim 10$ deg^{-2}), we estimate that the number of quasars in this color range is $\lesssim 65$ deg^{-2} down to $(J + F)/2 = 22.15$ mag. This result is equivalent to an upper limit on the total number of quasars of 110 deg^{-2} if we assume (as is true in other catalogs) that $\sim 60\%$ of the quasars have colors corresponding to $B - V \leq 0.30$ mag.[7]

There is undoubtedly more in the Galaxy than is in our model. However, the simple model that we have used represents the star counts to an accuracy that is consistent with the available observations, ~ 0.2 mag in color for particular peaks and ~ 25% in the number of stars in a given color bin. The model must break down as we look to further distances or observe with greater accuracy, since we know the luminosity function varies from position to position in other galaxies. For faint disk stars, it is reasonable to expect that discrepancies with the simple model will appear at least at the 10% level. We know that there are fluctuations of that magnitude in other galaxies associated with spiral arms. It will be interesting to see if future observations reveal any evidence for a "pudgy" disk, i.e., one with a scale height between the typical value of 350 pc we find for the "old" disk and the 1450 pc assumed by Gilmore and Reid for their "fat" disk. A definite indication of a metallicity gradient in the spheroid field stars should appear when more fields are studied at faint apparent magnitudes and

[7]This number is about twice as large as our previously quoted upper limit since the Koo-Kron (1982) data for SA 68 contain about twice as many blue stars (23) as does the Kron (1980) data (12).

at different galactocentric distances. This gradient might manifest itself by a tendency for the spheroid stars near the galactic center (or the plane of the disk) to be better described by a 47 Tuc–like color-magnitude diagram, whereas spheroid stars at larger galactocentric distances (or further from the plane) may be better fitted by an M92-like color-magnitude diagram. This discrimination will require data that are reliable to better than 0.1 mag in color for stars fainter than about $V=19$. Combined spectroscopic and photometric observations should lead to a more accurate and detailed spheroid luminosity function than the schematic one that is shown in Figure 1.

We are grateful to our observational colleagues for their hard work which made this analysis possible. Many of our colleagues shared their insights and data with us prior to publication. We are particularly indebted to G. Gilmore, I. King, D. Koo, R. Kron, D. C. Morton, K. U. Ratnatunga, K. Tritton, T. Tyson, and D. Weistrop for generous communication of results and ideas. M. Fall greatly improved the paper by his comments on an early draft. We are grateful to an anonymous referee for many valuable comments and questions. This work was supported by the National Science Foundation grant no. PHY-8217352 and NAS 8-32902.



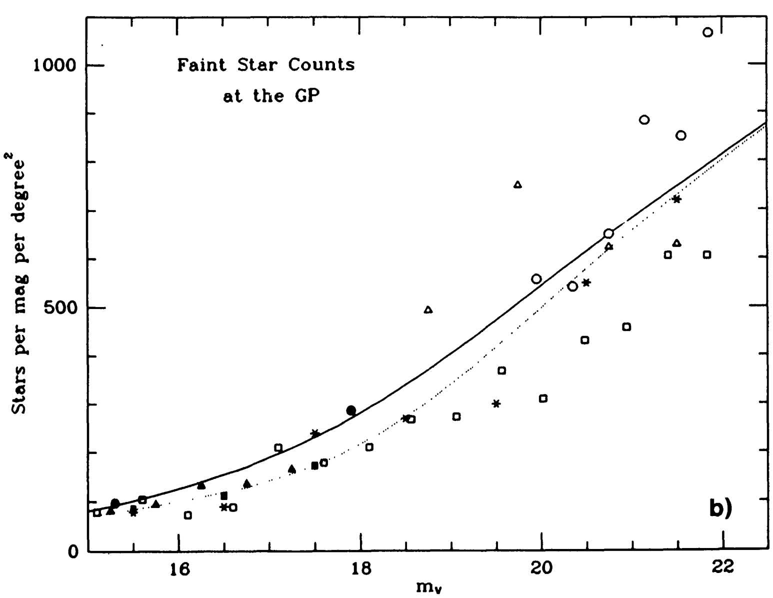


FIG. 21.—Predicted counts at the galactic pole with and without globular cluster feature (GCF). Full line is predicted by the standard model without the GCF; dotted line is obtained when the GCF is included. (a) Same as Fig. 4c; (b) same as Fig. 4d.

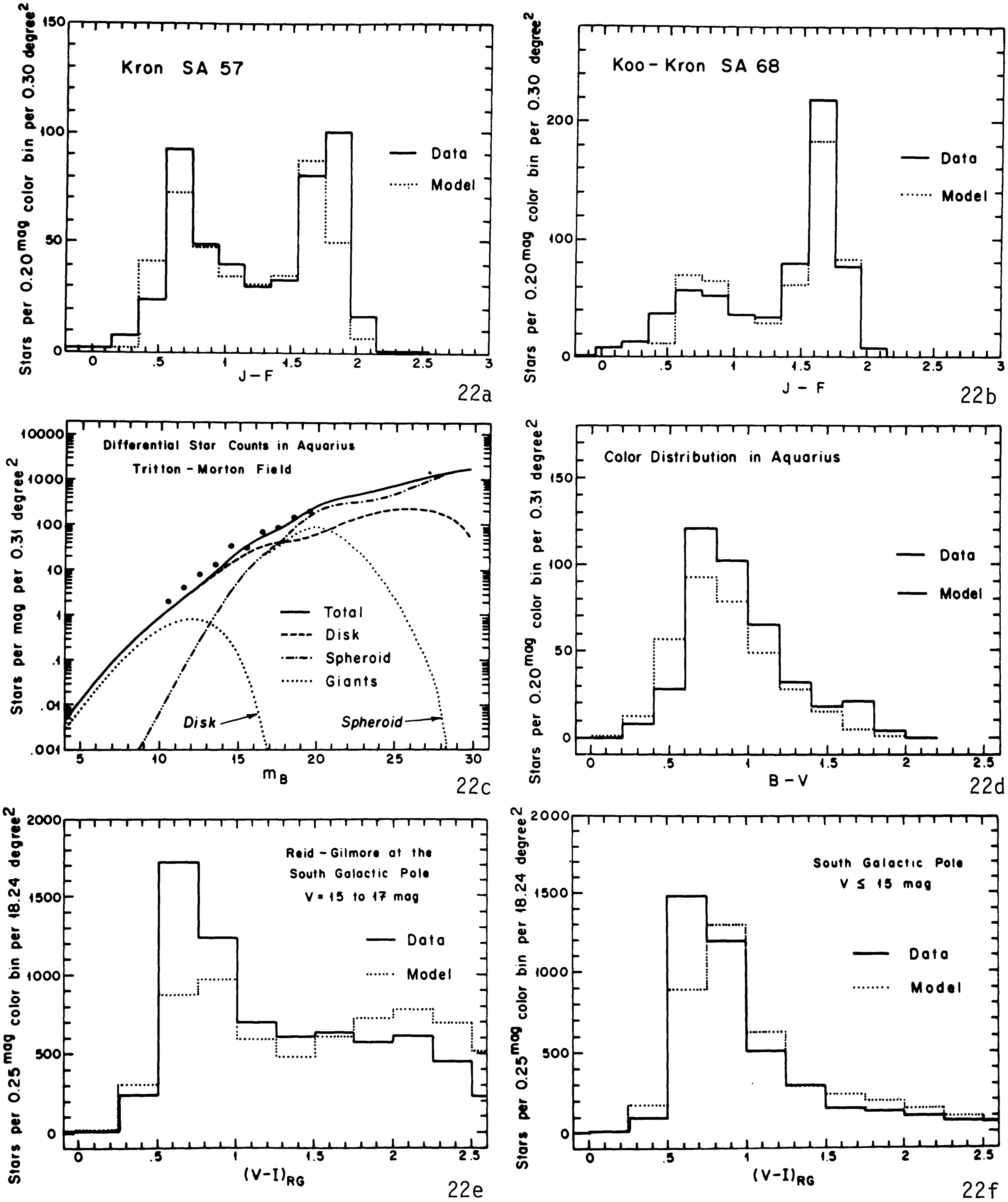


FIG. 22.—Comparisons with the distributions predicted by including the globular cluster feature (GCF). (*a*) Same as Fig. 5*b* (for SA 57); (*b*) same as Fig. 8*a* (for SA 68); (*c*)–(*d*) same as Figs. 12*a* and 12*c* (for the Tritton-Morton field); (*e*)–(*f*) same as Figs. 14*a* and 14*b* (for the south galactic pole).

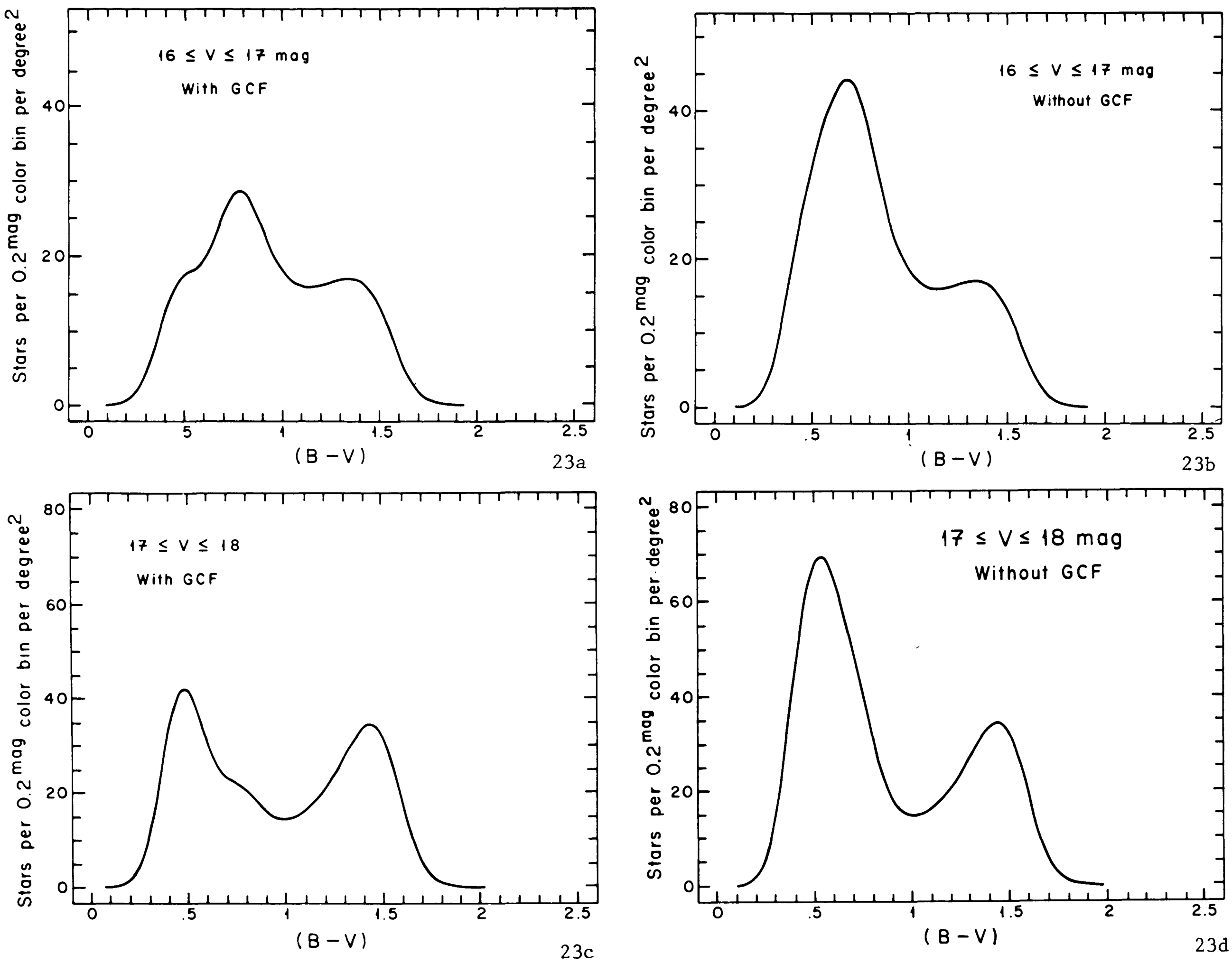


FIG. 23.—Proposed test for the globular cluster feature. (*a*)–(*d*) Comparison of color distributions predicted with and without the GCF at the galactic poles for $V=16$–18 mag. (*a*)–(*b*) $V=16$–17 mag; (*c*)–(*d*) $V=17$–18 mag.

APPENDIX A

GLOBULAR CLUSTER LUMINOSITY FUNCTION FEATURE NEAR $M_V \approx +1$ TO $+4$

We explore in this appendix whether the feature in globular cluster luminosity functions near $M_V \sim +1$ to $+4$ (see Fig. 1*a* where this feature is labeled "GCF" [globular cluster feature]) is present also in the luminosity function of the spheroid field population.[8] We have included the GCF and have repeated the calculations and the comparisons with observations that are described in the main text. We present here only the most informative results.

Figures 21*a* and 21*b* compare the differential star counts at the north galactic pole that were calculated—with and without the GCF—versus all the available observations. The maximum difference between the two calculated curves is of order 30%, which is comparable to the scatter among the various observational points (see also Fig. 3 and § II*c*).

The Kron (1980) data for SA 57 and the Koo-Kron data for SA 68 are fitted comparably well with and without the GCF (cf. Figs. 22*a* and 22*b* with Figs. 5*b* and 8*a*). There is a slight improvement in the fit to the observations for the Tritton-Morton field when the GCF is included (cf. Figs. 22*c* and 22*d* with Figs. 12*a* and 12*c*). The Reid-Gilmore (1982) color distributions are fitted better without the GCF (cf. Figs. 22*e* and 22*f* with Figs 14*a* and 14*b*).

We conclude that the available data are inadequate to decide whether or not the GCF exists in the luminosity function of the spheroid field stars.

The $B-V$ colors of stars in the approximate magnitude range $V \approx 16$–18 mag are a sensitive indicator of the GCF. Figures 23*a*–23*d* compare the predicted color distributions calculated with and without the GCF at the north galactic pole. Future wide

[8] Gilmore (1983) has claimed that there is strong evidence for this feature.

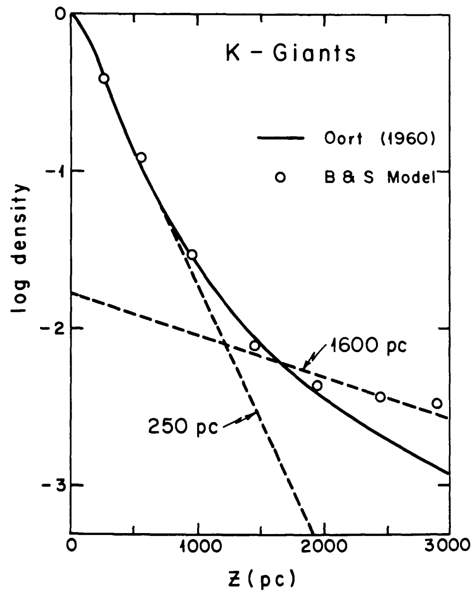


FIG. 24.—Distribution of K giants perpendicular to the galactic disk. Continuous curve is from Oort (1960), and open circles are densities calculated with the standard galaxy model. Two dashed straight lines are drawn in following Gilmore (1983) (see his Fig. 4). Dashed curve with the larger scale height indicates a thick disk—according to the arguments of Gilmore (1983)—although none is present in the model.

field observations at the galactic poles in B and V (with accurate photoelectric calibrations) between $V \approx 16$–18 mag could establish to what extent the GCF exists in the luminosity function of the spheroid field stars.

APPENDIX B

In a famous paper devoted to the study of K_z and of the local matter density, Oort (1960) analyzed the density distribution of K giants perpendicular to the galactic plane. Figure 24 compares the density of K giants in the standard galaxy model with Oort's summary of the data. We have used in the model calculations the absolute magnitude range of K giants given by Keenan (1963), namely, $-0.3 \leq M_V \leq +0.2$ mag. Our results are not sensitive to the precise range of absolute magnitudes that we adopt.

The gradual flattening of the density beyond 900 pc is caused, in the model, by spheroid stars. Beyond 1.2 kpc, the spheroid contributes more than half the calculated density.

The agreement between calculated and observed densities is excellent out to 2 kpc. Beyond 2 kpc, the model density decreases less rapidly than is indicated by Oort's summary of the data. It is not clear whether this is a fault in the model or in the data. The classification of low-metallicity (spheroid) K giants is known to be difficult with low-dispersion spectra like those used in the catalogs (HD and BSD) on which Oort's analysis was based.

Gilmore (1983) has argued that the observed K giant densities reported by Oort indicate the existence of a thick disk. In fact, our Figure 24 is essentially the same as his Figure 4, except that Gilmore did not include the predicted densities of the B&S model.

The density distributions in the standard galaxy model and in Oort's data are fitted well by two exponentials (see our Fig. 24 and Gilmore's Fig. 4), one with a scale height of ~ 250 pc and the other with a scale height of 1600 pc. These two "best-fit" exponentials are represented in Figure 24 by the two dashed lines. The intercept of the implied thick disk has, as stressed by Gilmore, 2% of the density of the thin disk at the galactic plane.

The thick disk that is implied by the dashed line in Figure 24 is not present in the standard galaxy model. Figure 24 shows that curve fitting can provide apparent evidence for a thick disk—with the properties suggested by Gilmore and Reid (1983) and Gilmore (1983)—in a physical situation in which none is present.

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